

# Constraints on the Size of Extra Compactified Dimensions from Compact Star Observations

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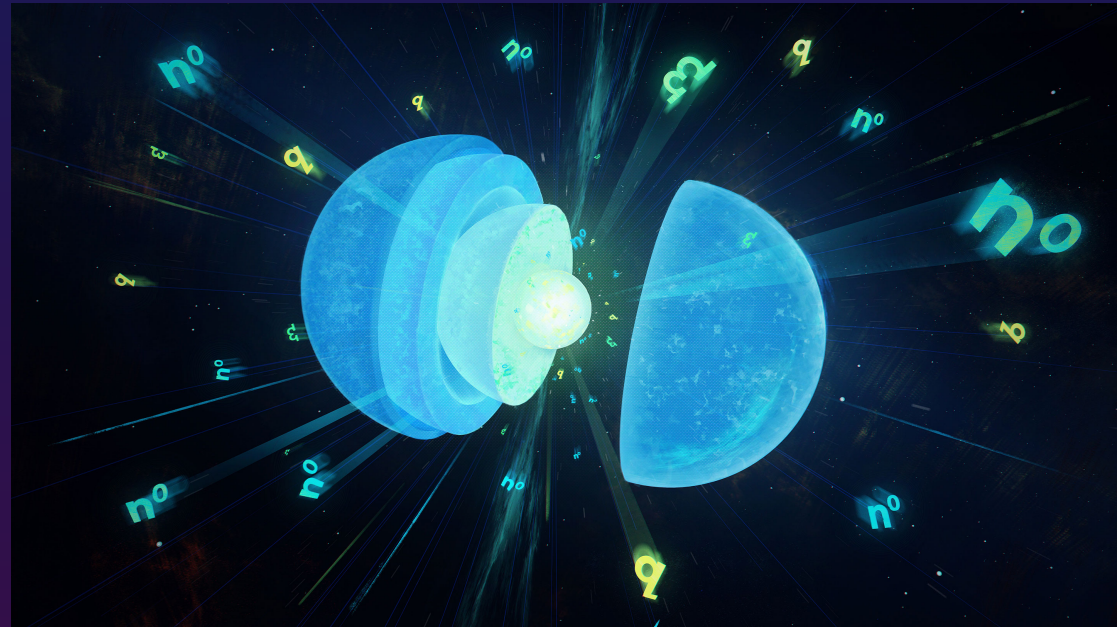
In collaboration with:

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*Emese Forgács-Dajka*

*Eötvös Loránd University*

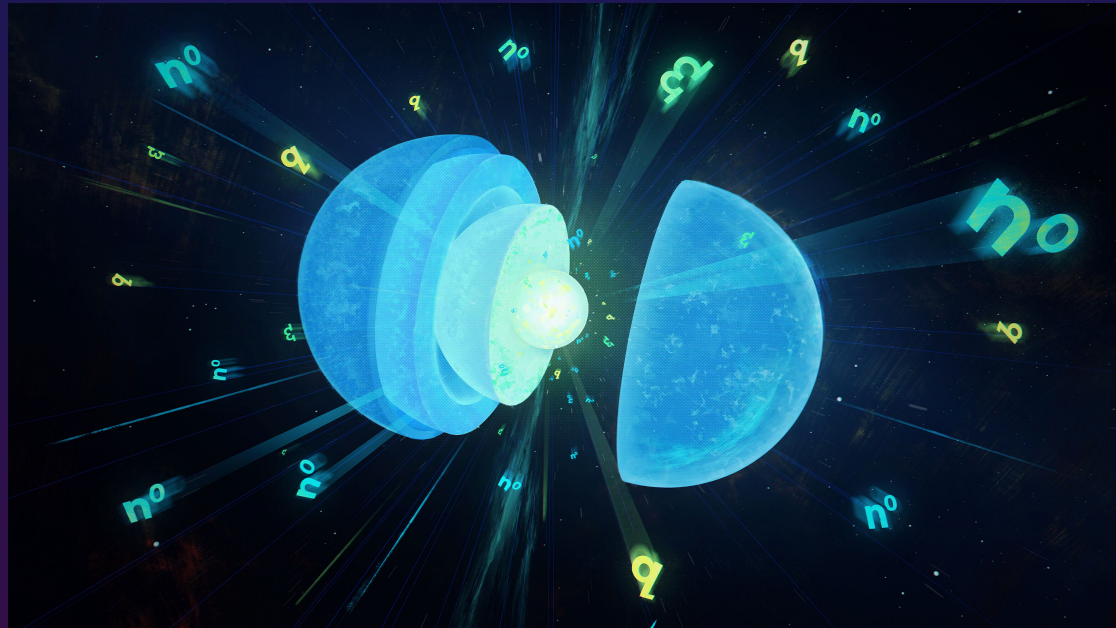


# Constraints on the Size of Extra Compactified Dimensions from Compact Star Observations

- Remnants of supernovae
- Supported by baryon degeneracy
- Compact objects:  
 $R \sim 10\text{km}$       $M \sim 1.1\text{--}2M_{\odot}$
- High density, low temperature



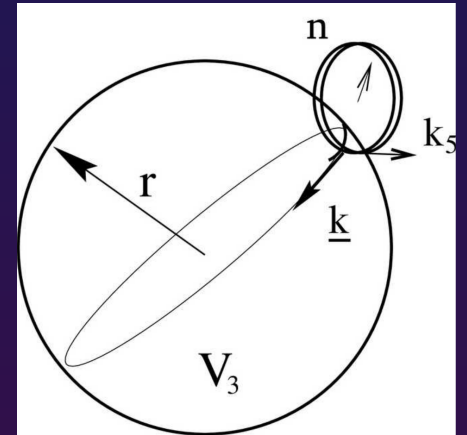
Probe physics in environments  
not available on Earth



# Treatment in $1+3+1_c$ dimensions

- Assume one extra **compactified spatial dimension** with size  $R_C$
- At **each point** in ordinary 3D space **particles** with enough energy **can move into it**
  - **3D**: particles with **different masses**
  - **3+1<sub>c</sub>D**: **one particle** but with **different quantized momenta** in the extra dimension

$$E_5 = \sqrt{\underline{k}^2 + \left(\frac{n}{R_C}\right)^2} + m^2 = \sqrt{\underline{k}^2 + \bar{m}^2}$$



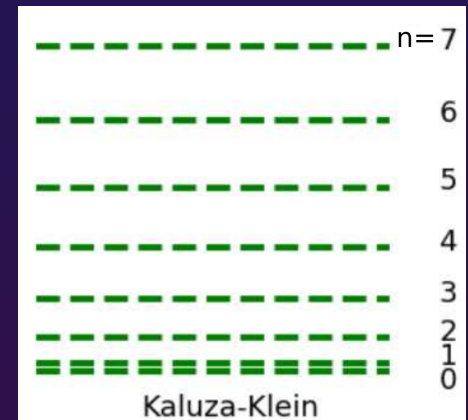
- With the **right** choice of  $R_C$  the **mass spectrum** of particles could be **reproduced**

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$$\bar{m}^2 = \left(\frac{n}{R_C}\right)^2 + m^2$$



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# Building up stars

- Two equations needed:
  - **Tolmann-Oppenheimer-Volkoff (TOV)** ~ GR hydrostatic equation

$$\frac{dp(r)}{dr} = -\frac{GM(r)\varepsilon(r)}{r^2} \times \left[1 + \frac{p(r)}{\varepsilon(r)}\right] \left[1 + \frac{4\pi r^3 p(r)}{M(r)}\right] \left[1 - \frac{GM(r)}{r}\right]^{-1}$$

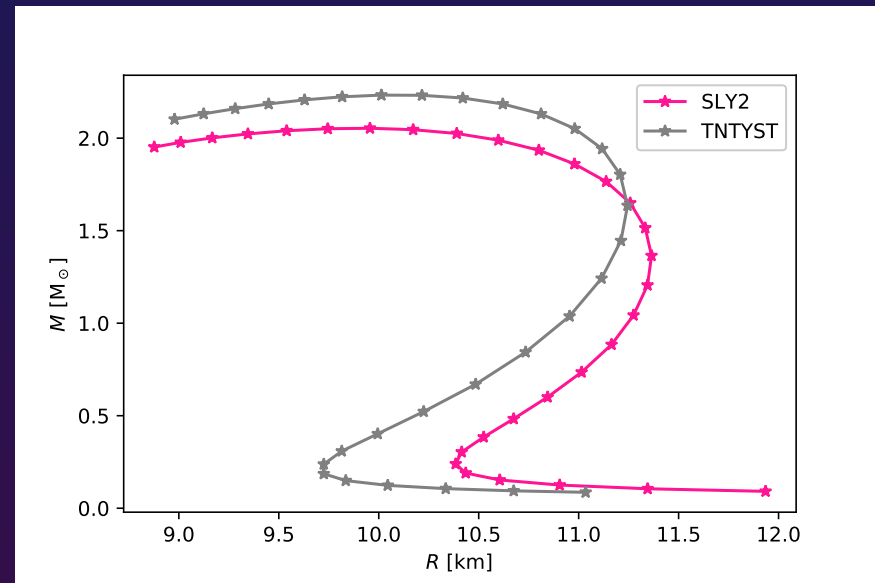
static,  
spherically  
symmetric

$$M(r) = \int_0^r dr' 4\pi r'^2 \varepsilon(r')$$

- **Equation of state (EoS)**  $\varepsilon(p)$
- **Boundary conditions:**
  - **Pressure at the surface**  $p(R) = 0$   
(in practice  $p(R) = p_{\min} = 10^{-5} \text{ km}^{-2}$ )
  - **Central energy density**  $\varepsilon_C$



*M-R diagrams*

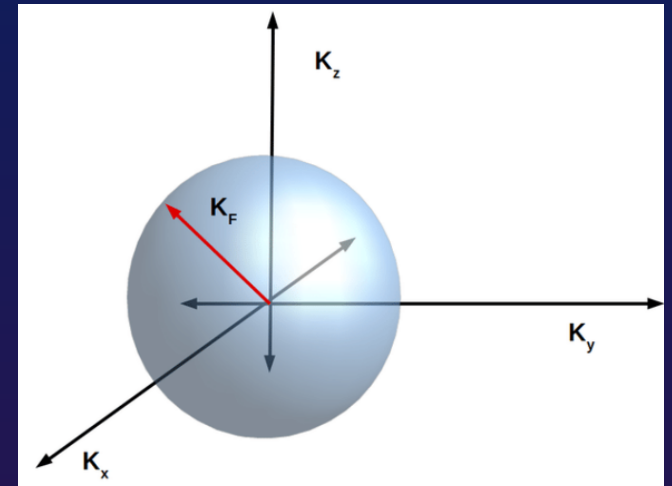


# EoS in 1+4D

- **Interacting** degenerate **Fermi gas**
- **Potential** is a **linear** function of density:  

$$U(n) = \xi n \quad \xi = \text{const}$$
- **Thermodynamic potential** on  $T=0$  MeV

$$\tilde{\Omega} = \frac{-2k_B T V_{(d)}}{h^d} \int \ln \left( 1 + e^{\frac{\mu - E(\mathbf{p})}{k_B T}} \right) d^d \mathbf{p}$$



- **Extra dimension**  $\longrightarrow$  calculate with **excited mass**
- **Interaction**  $\longrightarrow$  **chemical potential shifted by  $-U(n)$**

$$\epsilon(\mu) = \epsilon_0(\mu - U(n)) + \epsilon_{int}$$

$$p(\mu) = p_0(\mu - U(n)) + p_{int}$$

$$n(\mu) = n_0(\mu - U(n))$$

$$\epsilon_{int} = p_{int} = \int U(n) dn = \int \xi n dn = \frac{1}{2} \xi n^2$$

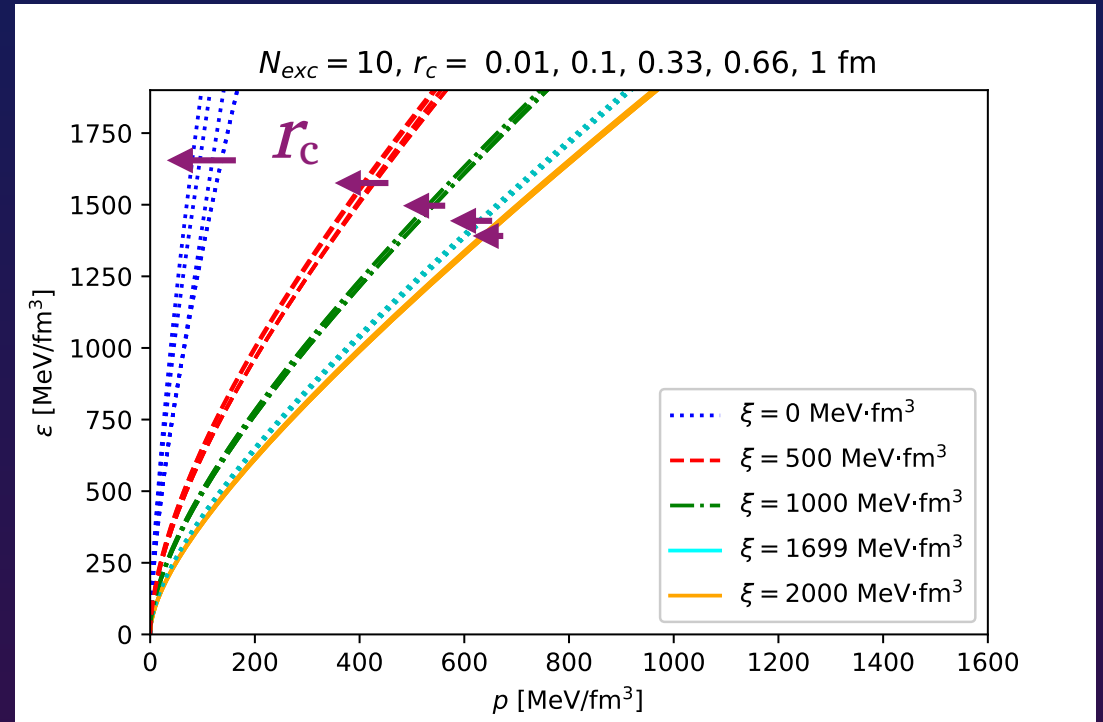
# Relativity in 1+4D

- Assume (for **TOV**):
  - ◊ Spherical symmetry
  - ◊ Time-independence
  - ◊ Isotropic relativistic ideal fluid
- Assume (for **extra dimension**):
  - Microscopic
  - ◊ 4D metric does not depend on  $g_{55}$
  - ◊ Causality postulates hold
  - ◊ Full Killing symmetry

$$g_{\mu\nu} = \begin{bmatrix} g_{00} & \cancel{g_{01}} & 0 & 0 & \cancel{g_{05}} \\ \cancel{g_{01}} & g_{11} & 0 & 0 & \cancel{g_{15}} \\ 0 & 0 & g_{22} & 0 & 0 \\ 0 & 0 & 0 & g_{22} \sin^2 \vartheta & 0 \\ \cancel{g_{05}} & \cancel{g_{15}} & 0 & 0 & \boxed{g_{55}} \end{bmatrix}$$

# Equation of state

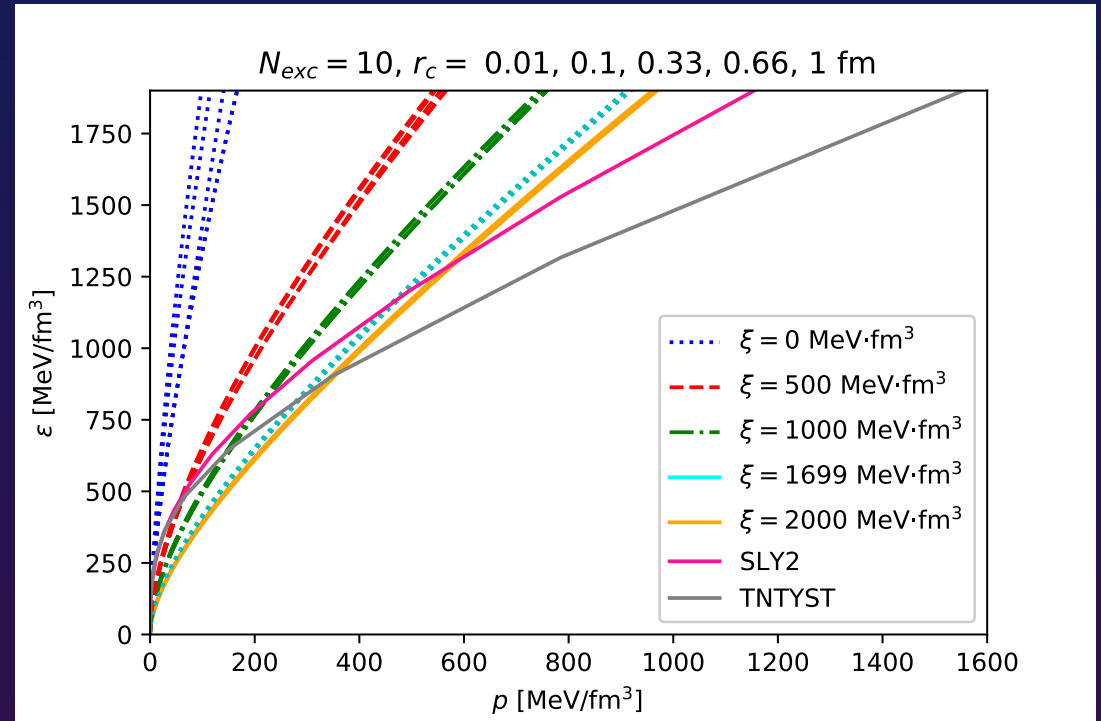
- $\xi$  dependence is much more dominant than  $r_c$
- The bigger  $\xi$ , the less important  $r_c$  becomes





# Equation of state

- $\xi$  dependence is much more dominant than  $r_c$
- The bigger  $\xi$ , the less important  $r_c$  becomes
- For lower energies small  $\xi$  approximates more refined nuclear matter EoSs
- For high energies a large  $\xi$  is a better approximation

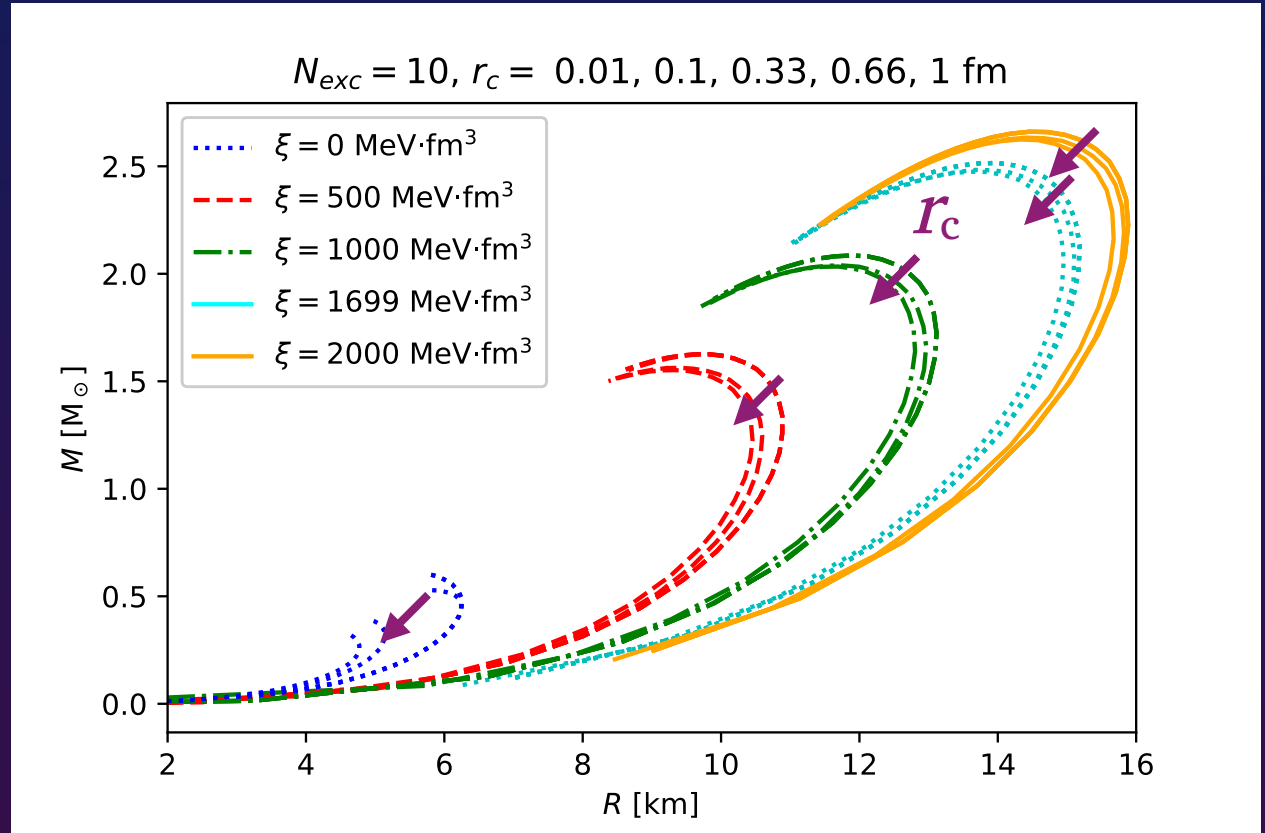


<https://compose.obspm.fr/>

1. H. Togashi, K. Nakazato, Y. Takehara, S. Yamamuro, H. Suzuki and M. Takano, Nucl. Phys. A 961 (2017) 78
1. F. Gulminelli and Ad. R. Raduta, arXiv:1504.04493.
2. E. Chabanat, Ph.D. thesis, University Claude Bernard Lyon-1, Lyon, France, 1995.
3. P. Danielewicz et J. Lee, Nucl. Phys. A818, 36 (2009).

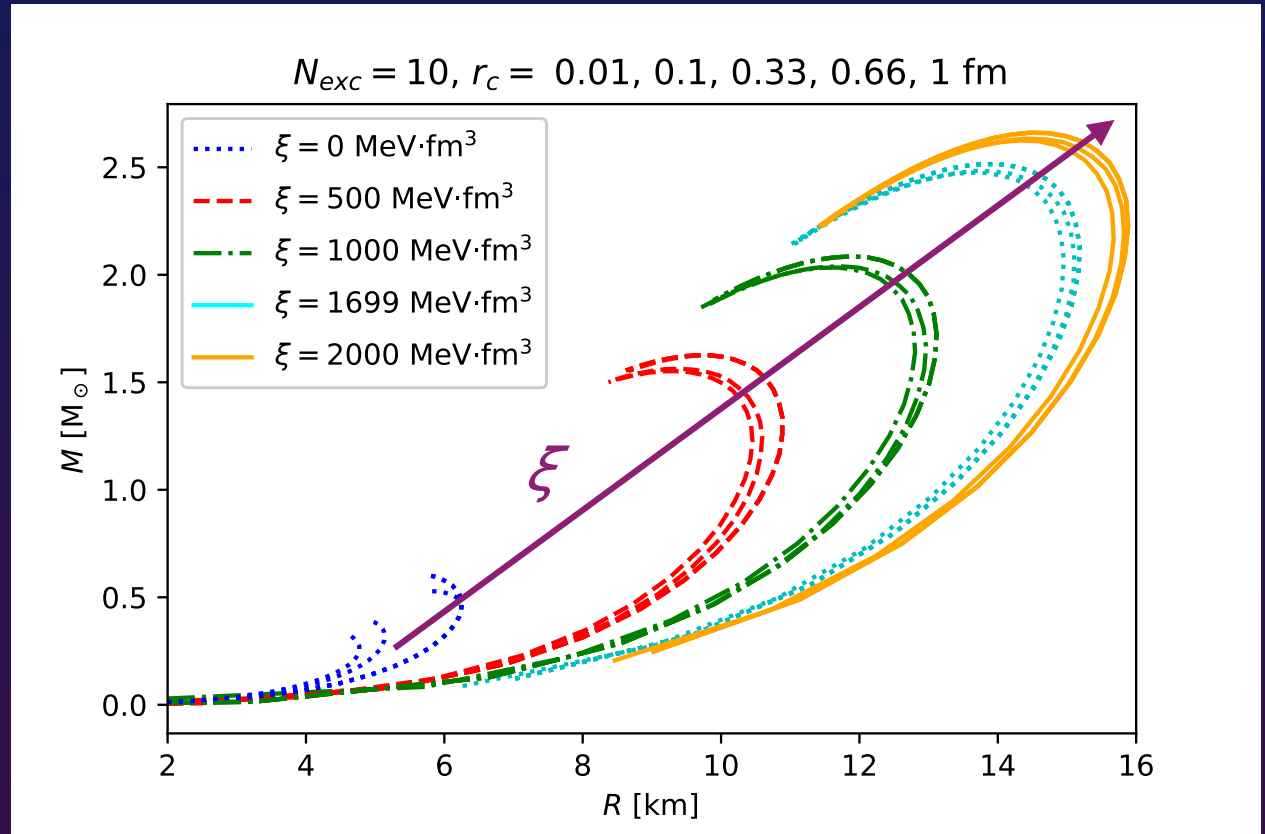
# $M$ - $R$ diagrams of the EoS

- $\xi$  dependence is much more dominant than  $r_c$  (latter only  $\sim 5\%$ )
- The bigger  $\xi$ , the less important  $r_c$  becomes



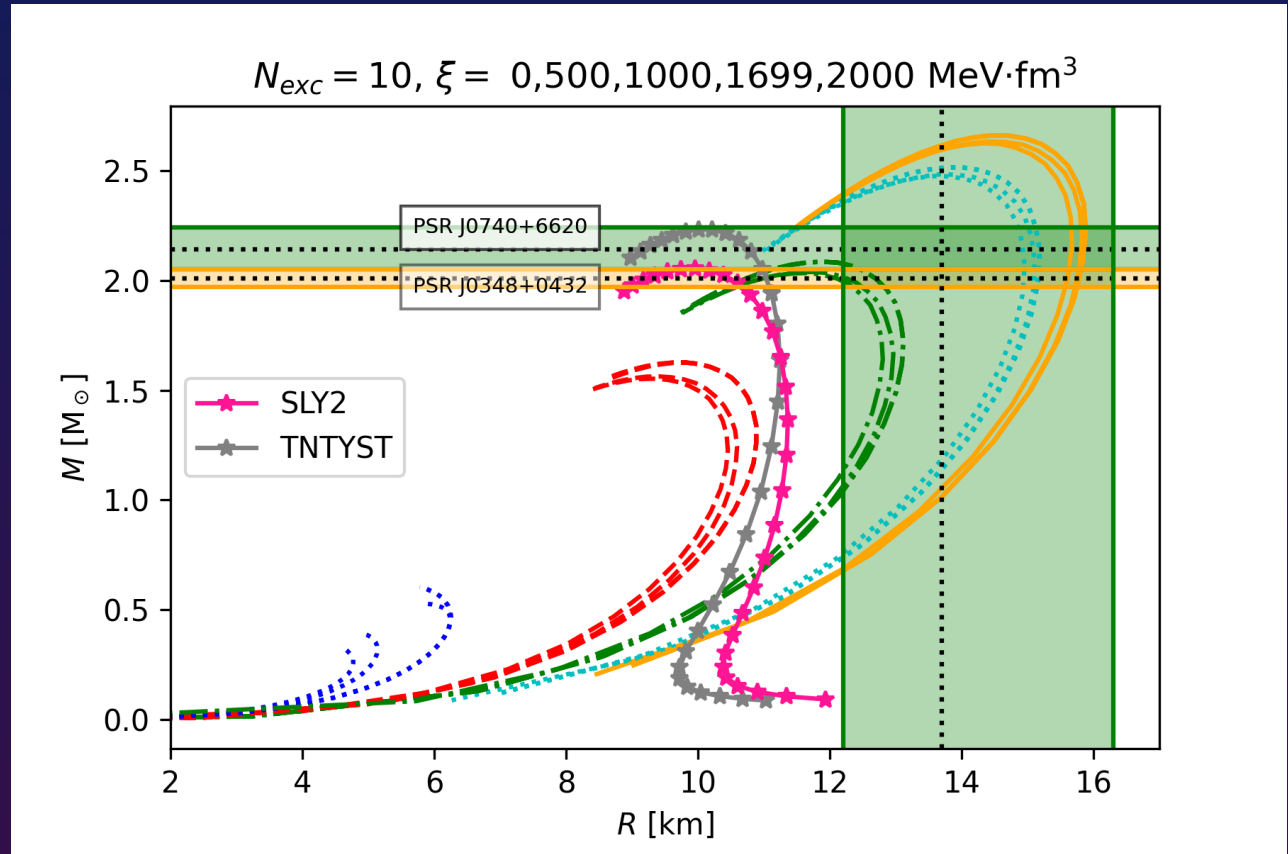
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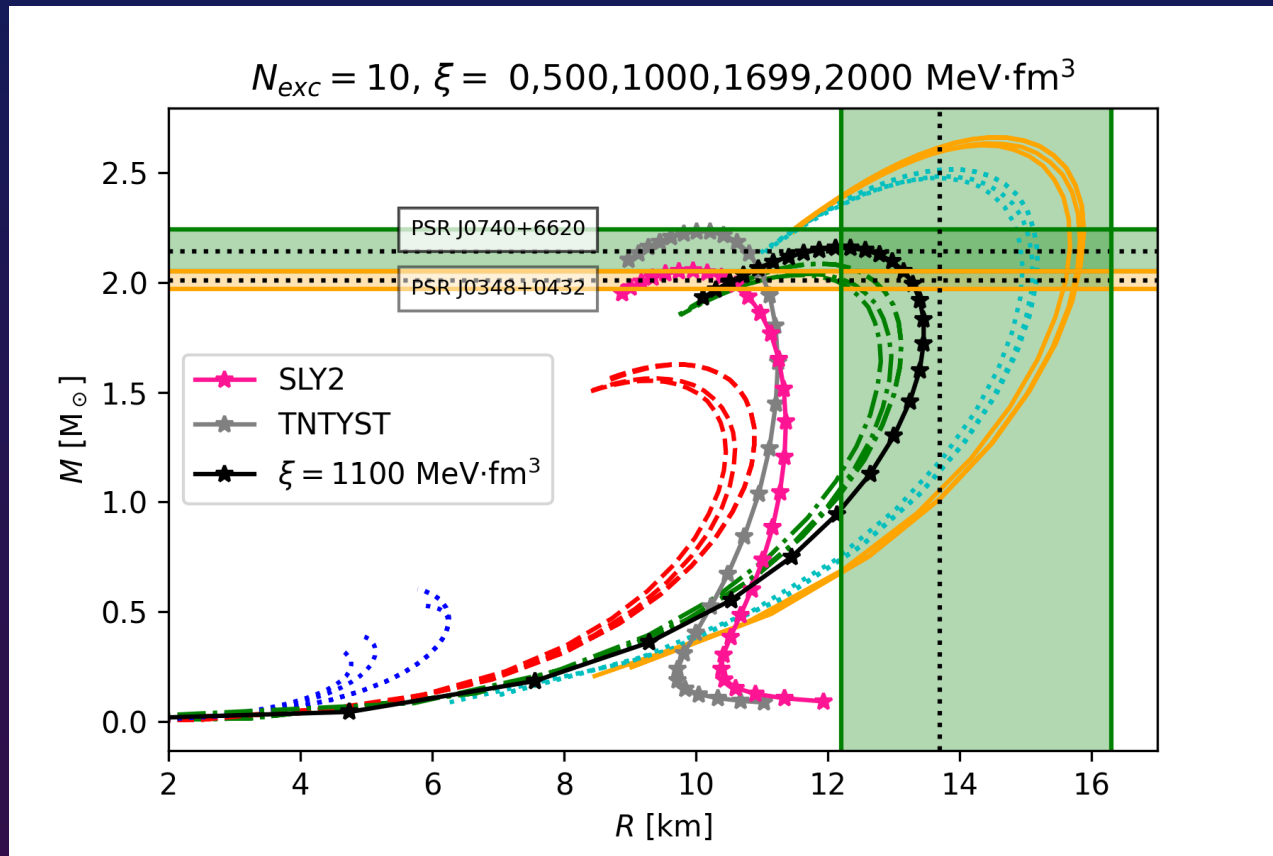
# $M$ - $R$ diagrams

- + measurement data
- + 2 more refined EoSs



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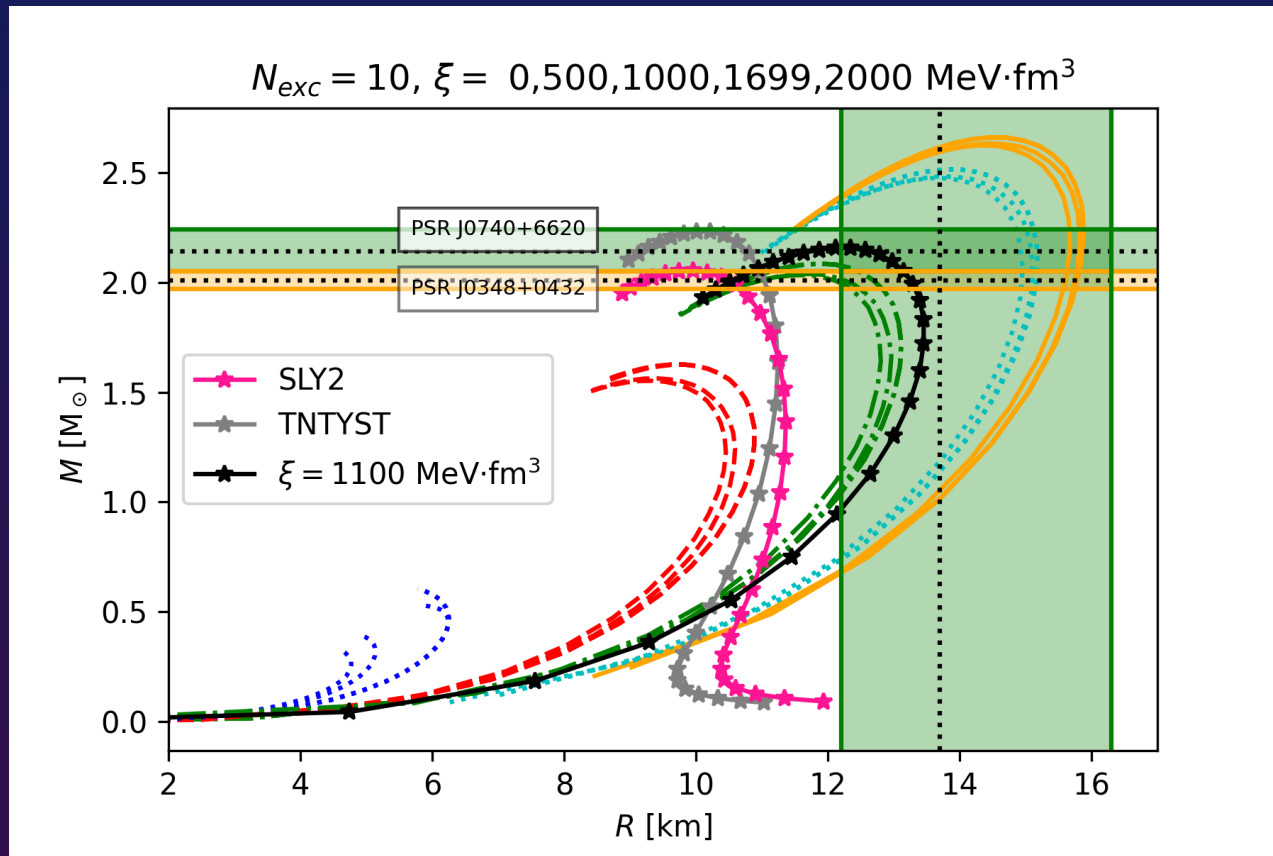
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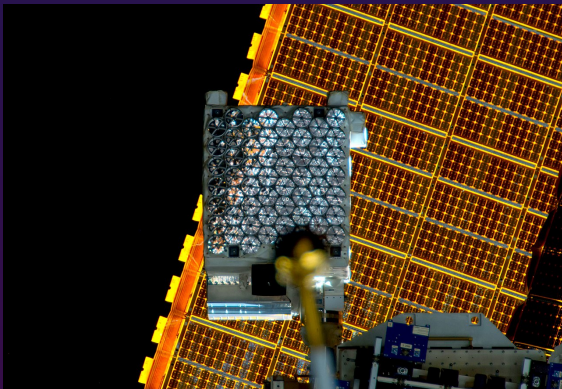
Change position of black curve by **setting** the **size** of the **extra dimension**.



# Summary



LIGO



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- Model with the possibility of probing **beyond standard model physics**
- One **extra spatial compactified dimension**

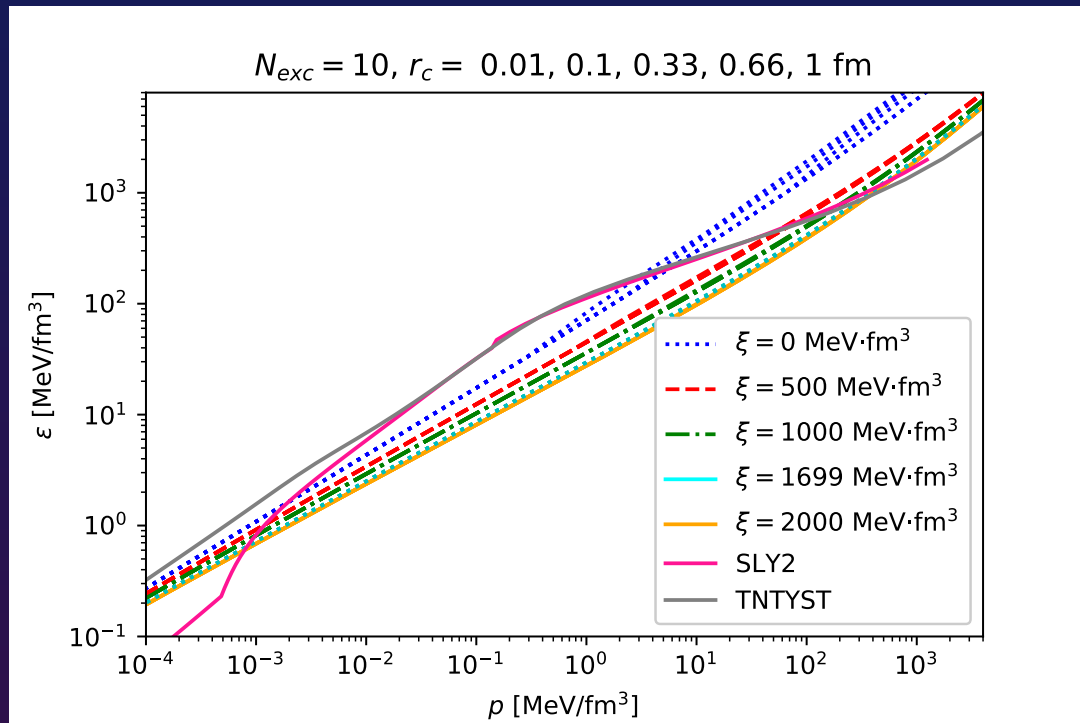


Ordinary **mass** can be described as **quantized 5<sup>th</sup>D momenta**

- Effective nuclear field theory with **linear repulsive potential**
- It is **possible** to **build** compact stars with **realistic** properties
- **Constraints** on the **size of possible extra dimensions** could be given using more precise **observational data**

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