# Hadron fragmentation in the non-extensive statistical approach

**G.G. Barnaföldi** & Á. Takács & G. Kalmár

Wigner Research Centre for Physics

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*References: EPJA 55(2019) 126, Universe 5 (2019) 122; 134;*

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# Outline

- Motivation
  - The non-extensive phenomena: Tsallis–Pareto distribution
  - Spectra fit in high-energy collisions

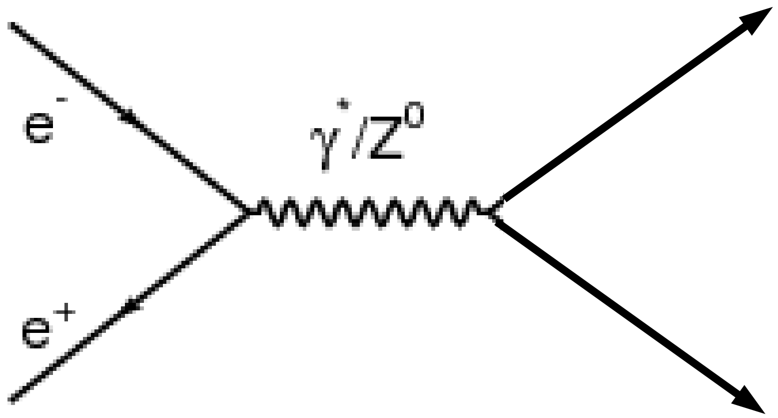
- Non-extensive fragmentation function parametrization in $e^+e^-$
  - A statistical model for hadron production in $e^+e^-$ collisions
  - A non-extensive, Tsallis-like fragmentation function parametrization
  - Validity of scaling and comparison to other FFs.

- Discussion
  - Hadronization in the non-extensive statistical approach
  - Connection to the ‘Tsallis thermometer’

# Motivation for the non-extensive hadronization models

# Modeling hadronization in $e^+e^-$ collisions

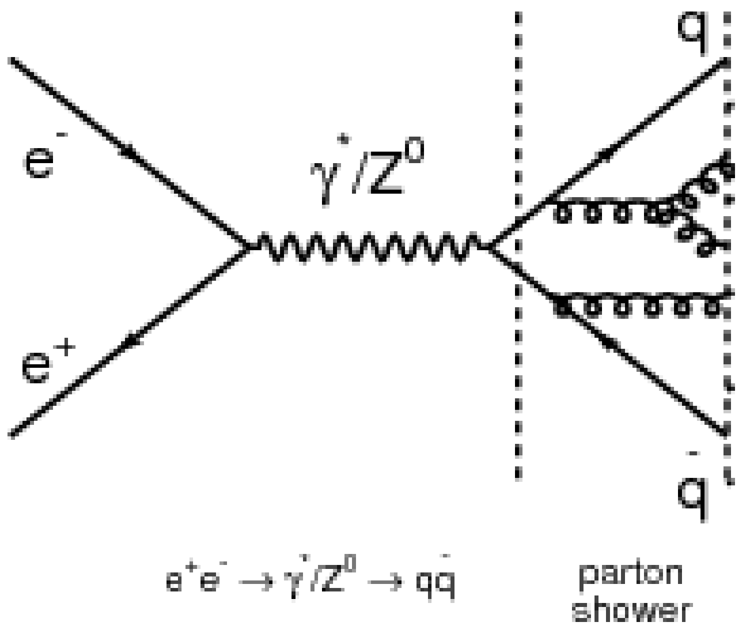
Final state processes & hadronization




$e^+e^- \to \gamma^*/Z^0 \to q\bar{q}$

# Modeling hadronization in $e^+e^-$ collisions

Final state processes & hadronization



# Modeling hadronization in $e^+e^-$ collisions

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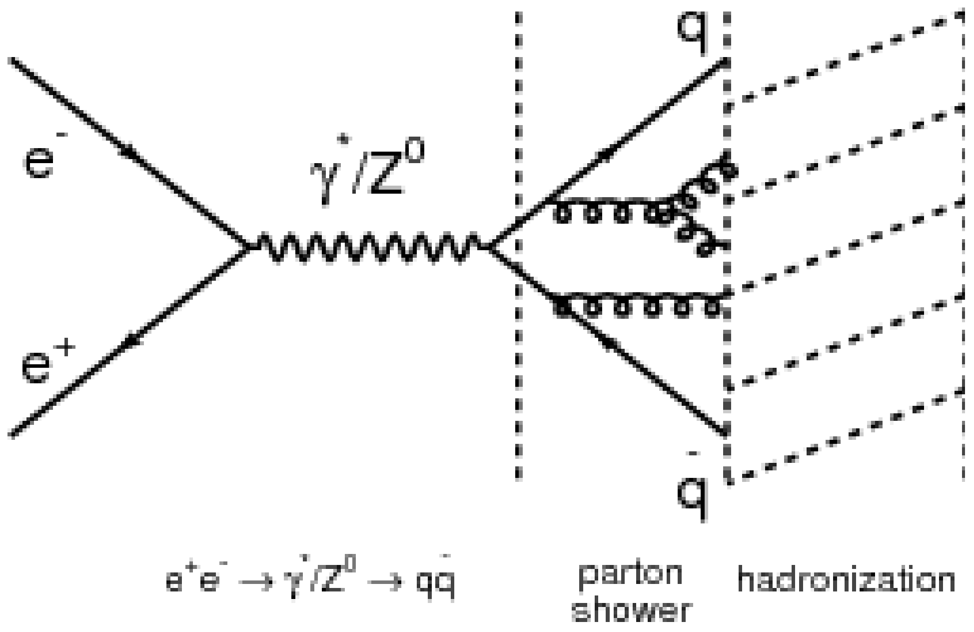
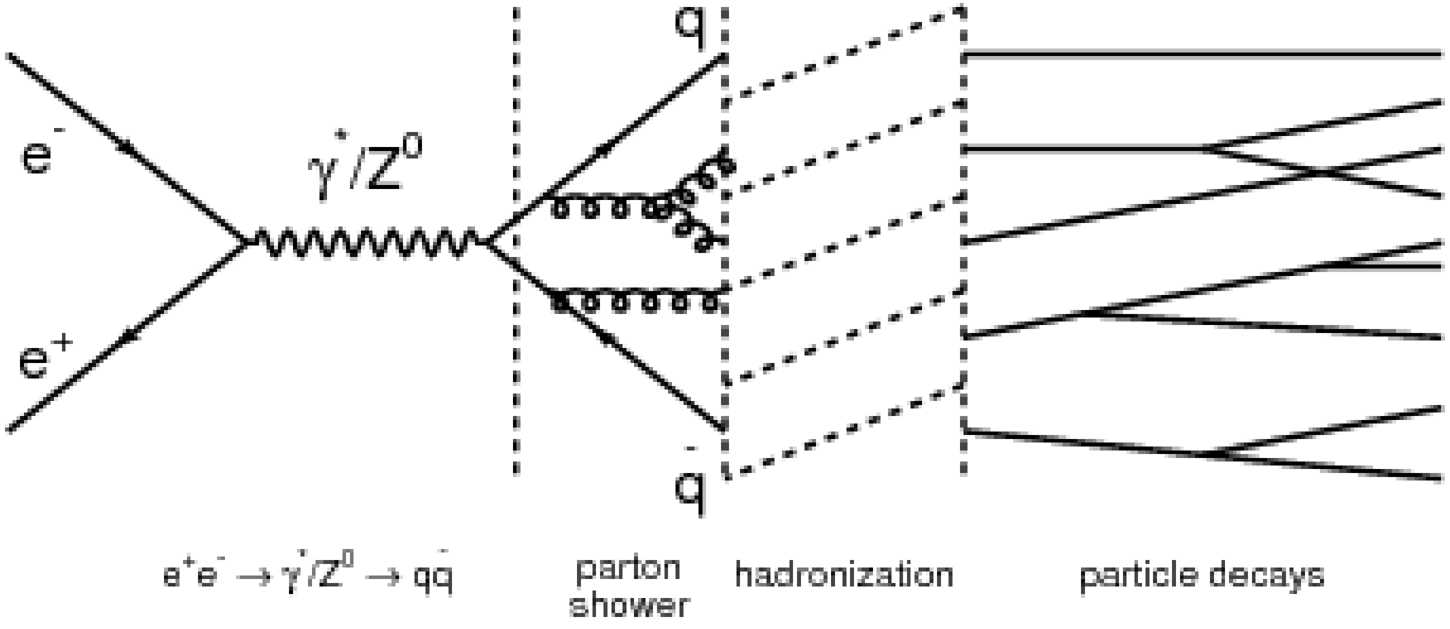


# Modeling hadronization in $e^+e^-$ collisions

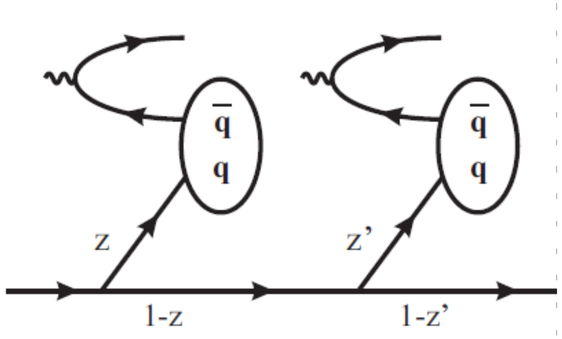
Final state processes & hadronization



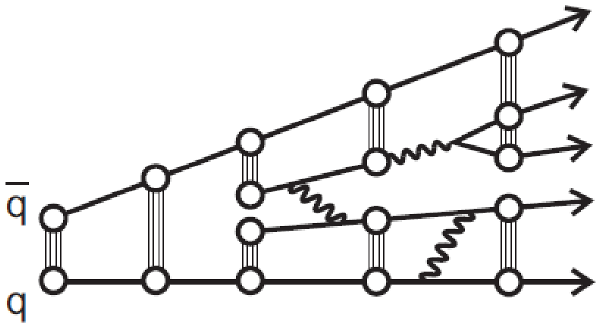
# Hadronization models – history

The hadronization models so far…

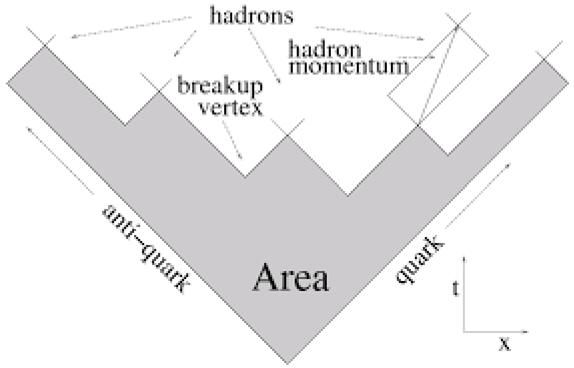
Feynman-Field



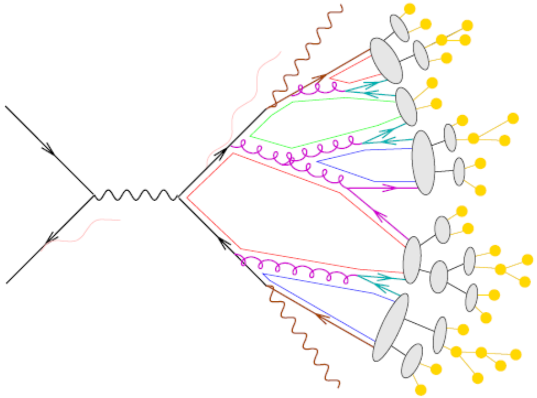

pair production




Lund model




cluster model



$$f(z) \propto \left[ z \left( 1 - \frac{1}{z} - \frac{\epsilon}{1-z} \right)^2 \right]^{-1}$$

$$f(z) \propto z^{-1}(1-z)^a \cdot \exp\left( \frac{-b\, m_T^2}{z} \right)$$

pQCD models

Non-pQCD models

PYTHIA/HIJING

HERWIG

# Motivation for the non-extensive formula

Can we make the next evolution step?

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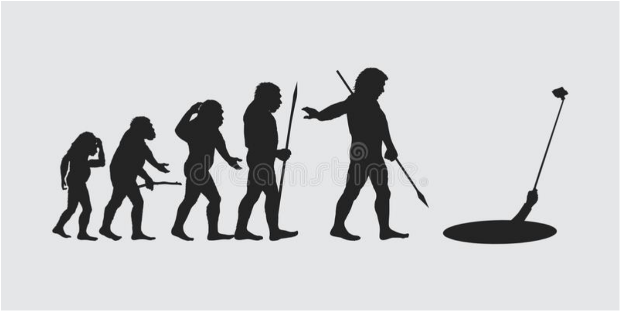


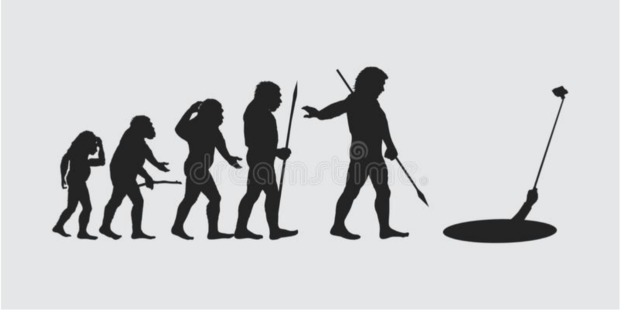
# Motivation for the non-extensive formula

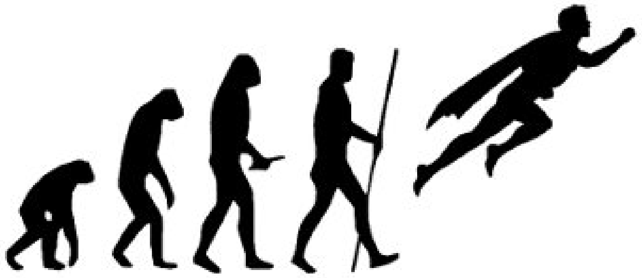
Can we make the next evolution step?

Stay on the treadmill: use old ansatz with new HI data, even that is not ‘clean’ $e^+e^-$ data.

Hide to the unknown: use machine learning to find the best-fit ansatz with NO physics.

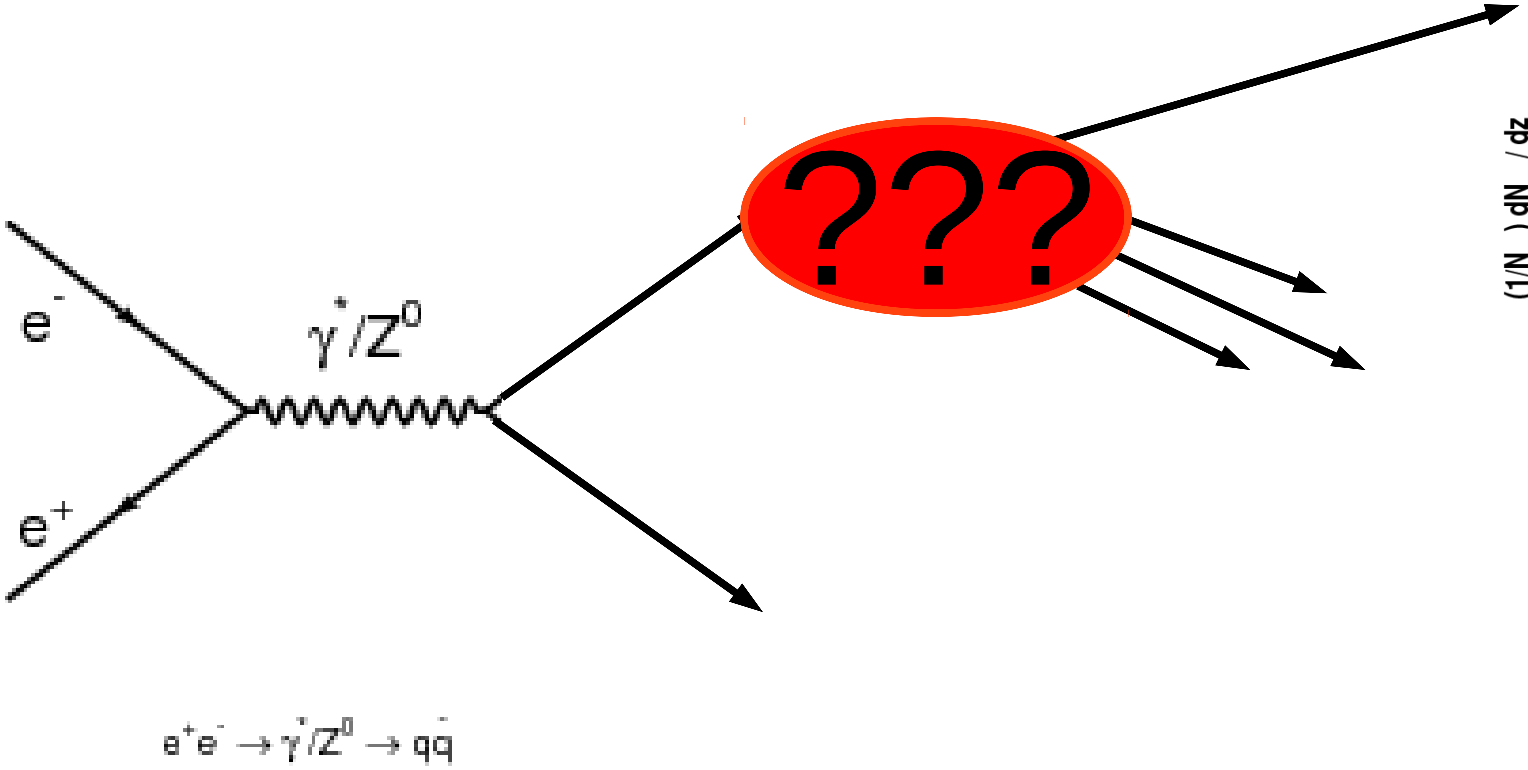
Make a real evolution step: physics-motivated anzatz, with parameter-evolution & predicted values .

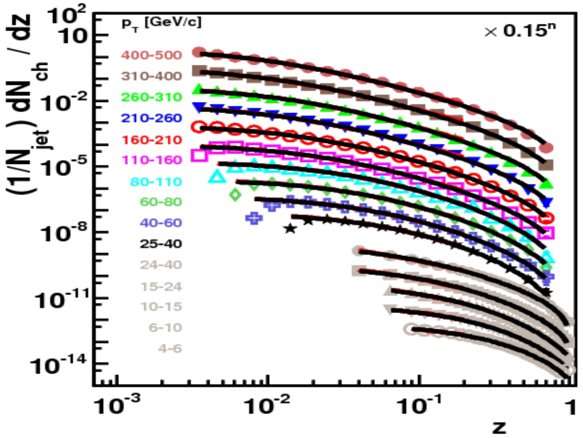




# Motivation for the non-extensive formula

Hadronization in the phenomenologial picture



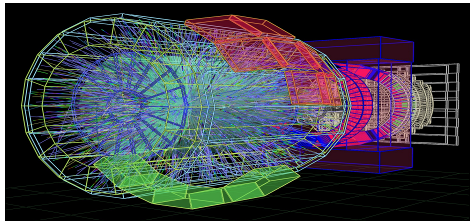



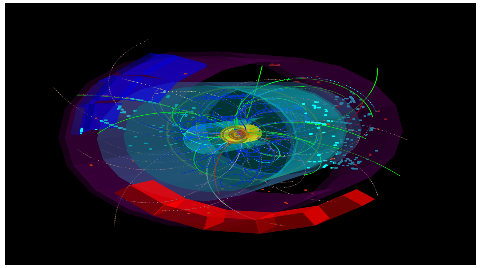

Ürmössy K, BGG, Biró TS
PLB701 (2011) 111
PLB718 (2012) 125

# Motivation for the non-extensive formula

- Non-extensive statistics: Tsallis – Pareto distribution

$$f(\varepsilon) = \left[1 + (q-1)\frac{\varepsilon}{T}\right]^{-\frac{1}{q-1}} \quad \leftarrow \quad S_{12} = S_1 + S_2 + (q-1)S_1S_2$$





$$q = \frac{\langle S'(E)^2 + S''(E)\rangle}{\langle S'(E)\rangle^2}$$

$$\frac{1}{T} = \langle S'(E)\rangle$$

$$q = \frac{\langle n(n-1)\rangle}{\langle n\rangle^2}$$

$$q = 1 + \frac{\Delta T^2}{T^2} - \frac{1}{C}$$

$$T = \frac{E}{\langle n\rangle}$$

$$T = \frac{\int \epsilon f_{TS}(\epsilon)}{\int f_{TS}(\epsilon)} = \frac{DT}{1-(q-1)(D+1)}$$

Eur. Phys. J. A49 (2013) 110,
Physica A 392 (2013) 3132

# Motivation for the non-extensive formula

| Name | Feynman-Field polynomial | String (Lund) model | Non-extensive (Tsallis-like) |
|---|---|---|---|
| Formula | $D_i^h(z,Q^2) = N_i^h z^{\alpha_i^h}(1-z)^{\beta_i^h}$ | $f(z) \propto z^{-1}(1-z)^a \cdot \exp\left(\frac{-b\,m_T^2}{z}\right)$ | $D_i^h(z,Q) = N_i^h(1-z)\left[1-\frac{q_i^h-1}{T_i^h}\log(1-z)\right]^{-\frac{1}{q_i^h-1}}$ |
| Physical motivation | propagator-like, power-law spectra | propagator-like + string model power-law + exponential | Non-extensive phenomena Tsallis-Pareto spectra |
| Physical meaning of parameters | No: spectra power, disagree with the theory | String tension + slope, but no for spectra power | Depending the statistical framework q (non-extensivity), T |
| Number of parameters | 3/channel | 3/channel | (2+1)/channel (normalized) |
| Evolution | DGLAP | DGLAP for power law | DGLAP |

# Fragmentation function parametrization in the non-extensive statistical approach

# Fit the non-extensive formula in $e^+e^-$ collisions

LO pQCD parton model

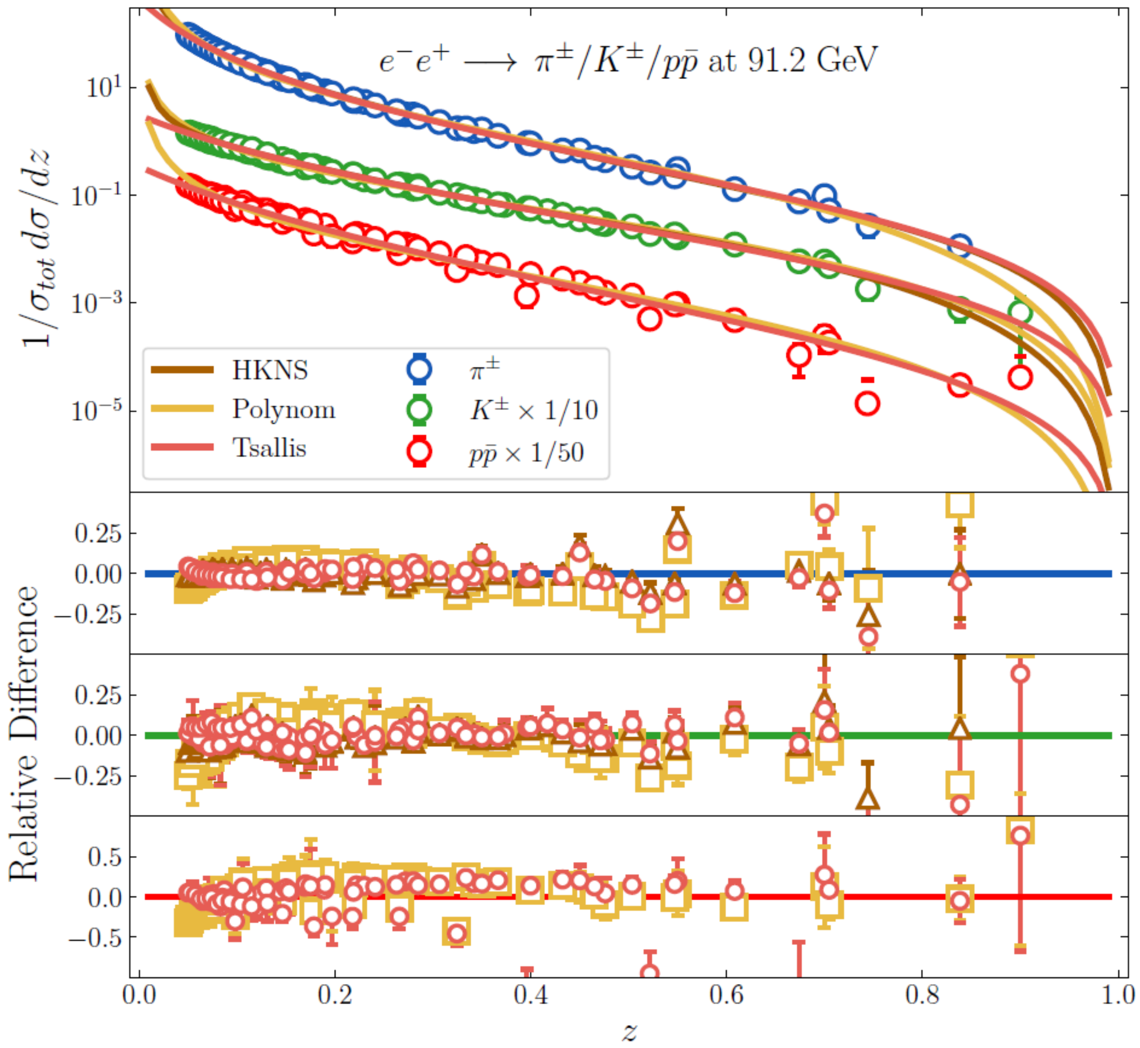
$$\frac{d\sigma(e^-e^+ \to hX)}{dz} = \sum_i \sigma_0^i(s) D_i^h(z,Q)$$

measure

parameters

Fit identified pion data in $e^+e^-$ collisions to get FF parameters.

Comparison to KKP, HKNS FFs



# Fit the non-extensive formula in $e^+e^-$ collisions

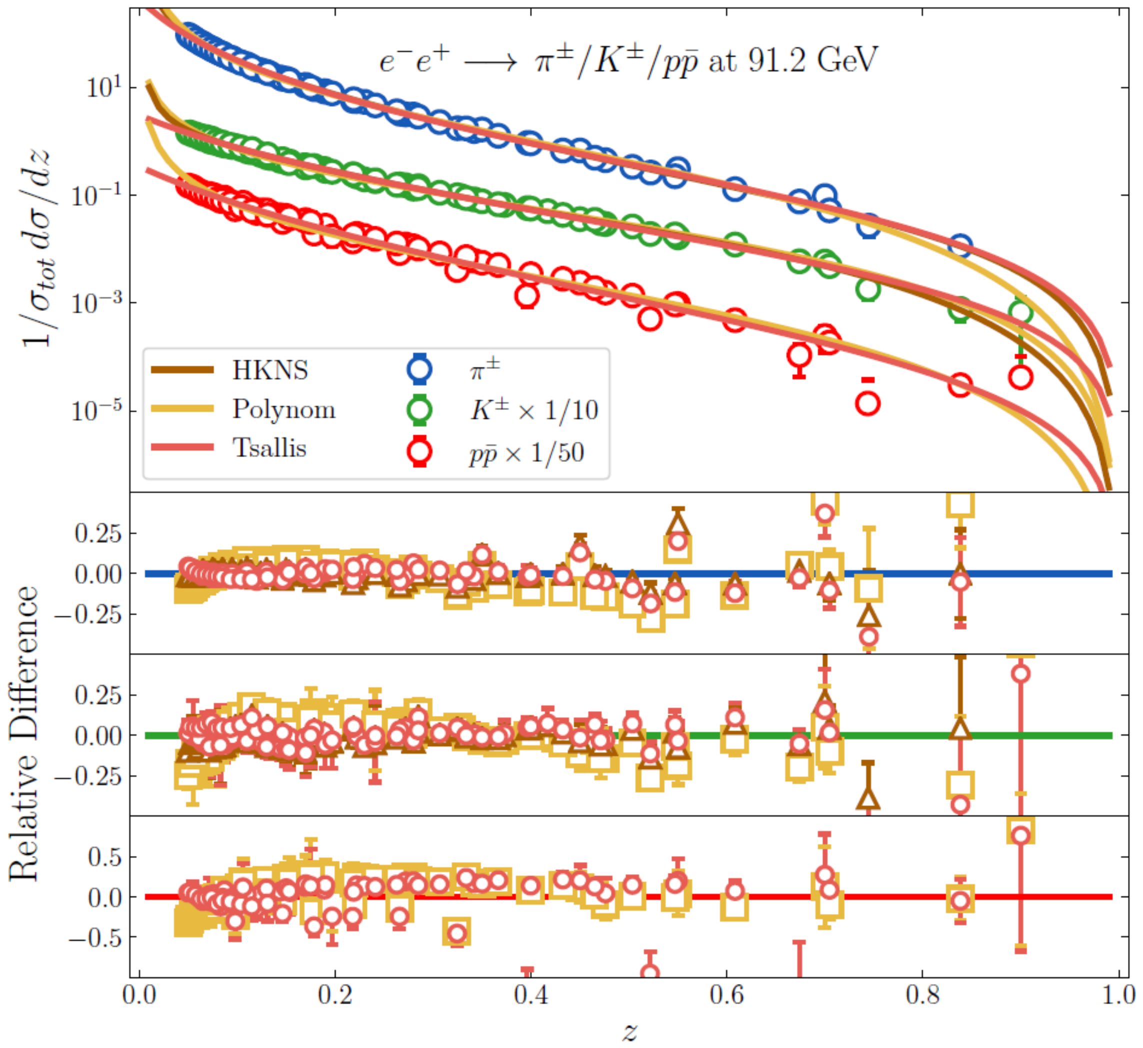
## LO pQCD parton model

Partonic channels q→h are fitted at initial Q scale in LO.

$$F^h(z,Q) = \frac{1}{\sigma_{tot}} \frac{d\sigma(e^-e^+ \to hX)}{dz}$$

via minimizing the merit function, using the data

$$\chi^2 = \sum_i \frac{\left(F^h(x_i,Q^2) - y_i\right)^2}{(\sigma_i)^2}$$



# Fit the non-extensive formula in $e^+e^-$ collisions

## LO pQCD parton model

Partonic channels q→h are fitted at initial Q scale in LO.

Need to reduce the number of fit parameters by the symmetries:

$$D_i^{\pi^0}(z,Q) = \frac{1}{2}\left[D_i^{\pi^+}(z,Q) + D_i^{\pi^-}(z,Q)\right]$$

$$\pi^+ = |u\bar{d}\rangle$$
$$\pi^- = |\bar{u}d\rangle$$

$$D_q^{\pi^-}(z,Q) = D_{\bar{q}}^{\pi^+}(z,Q)$$
$$D_g^{\pi^-}(z,Q) = D_g^{\pi^+}(z,Q)$$

$$D_u^{\pi^+} = D_{\bar{d}}^{\pi^+},$$
$$D_d^{\pi^+} = D_{\bar{u}}^{\pi^+} = D_s^{\pi^+} = D_{\bar{s}}^{\pi^+},$$
$$D_c^{\pi^+} = D_{\bar{c}}^{\pi^+},$$
$$D_b^{\pi^+} = D_{\bar{b}}^{\pi^+},$$
$$D_g^{\pi^+}.$$

For charge-average pions

# Fit the non-extensive formula in $e^+e^-$ collisions

## LO pQCD parton model

Partonic channels q→h are fitted at initial Q scale in LO.

Need to reduce the number of fit parameters by the symmetries:

Isospin, (anti)particle, neglect top, sea/valence contributions

(2×6+1)×3=69 → 3×(2×2+1)=15

$$\pi^+ = |u\bar{d}\rangle$$
$$\pi^- = |\bar{u}d\rangle$$
$$D_q^{\pi^-}(z,Q) = D_{\bar{q}}^{\pi^+}(z,Q)$$
$$D_g^{\pi^-}(z,Q) = D_g^{\pi^+}(z,Q)$$

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$$D_g^{\pi^+}.$$

For charge-average pions

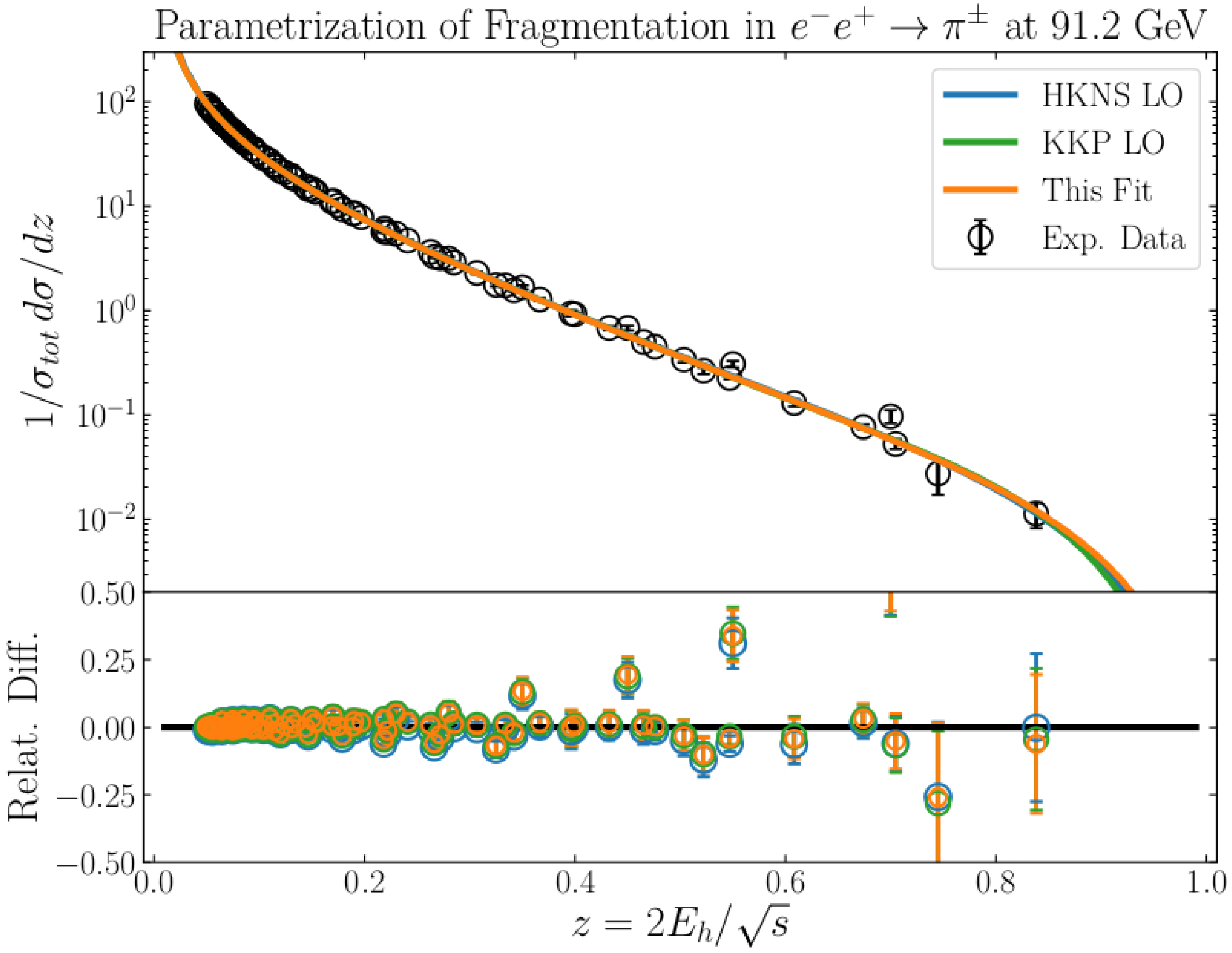
# Fit the non-extensive formula in $e^+e^-$ collisions

## LO pQCD parton model

Partonic channels q→h are fitted at initial Q scale in LO.

We used the symmetries of sea & valence channels, up to beauty.

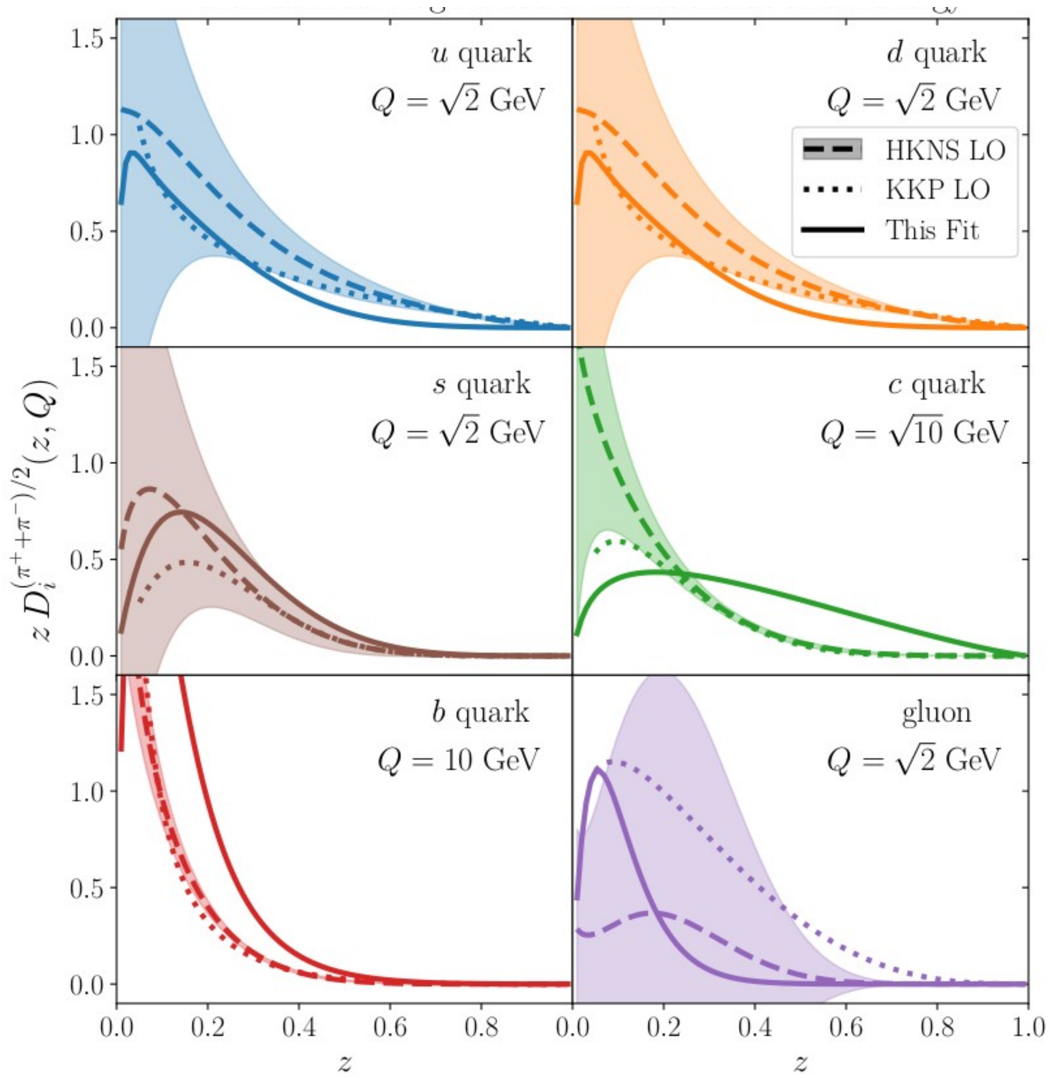
$$D_i^{\pi^0}(z,Q) = \frac{1}{2}\left[D_i^{\pi^+}(z,Q) + D_i^{\pi^-}(z,Q)\right]$$



# Fit the non-extensive formula in $e^+e^-$ collisions

## Pion LO FF parametrization

- All FFs have similar high-z trend, especially for valence quarks
- Non-extensive FFs have a clear  maxima at low z (< 2GeV) values.
- KKP has low-z cut, HKNS presents uncertainties
- Sea quark & gluon channels has more difference.



# Fit the non-extensive formula in $e^+e^-$ collisions

## Parameter scale evolution

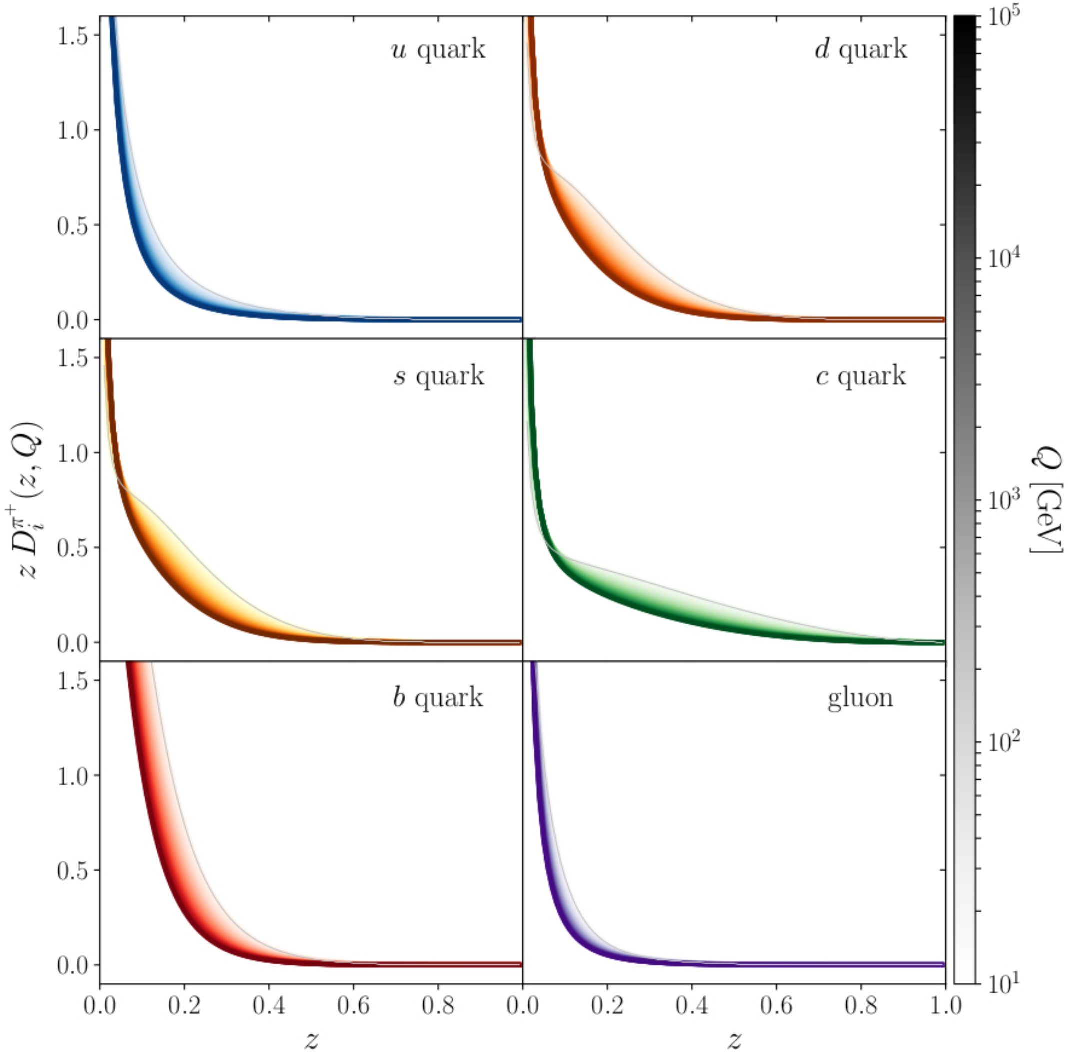
DGLAP evolution is given in LO:

$$\frac{dD_i^h(z,Q^2)}{d\log Q^2} = \sum_j \int_z^1 \frac{dx}{x} P_{ij}(z/x,Q^2) D_i^h(x,Q^2)$$

This is converted fitted by a simply

formula for the parameters

$$D_i^h(z,Q) = N_i^h (1-z) \left[1 - \frac{q_i^h - 1}{T_i^h} \log(1-z)\right]^{-\frac{1}{q_i^h - 1}}$$



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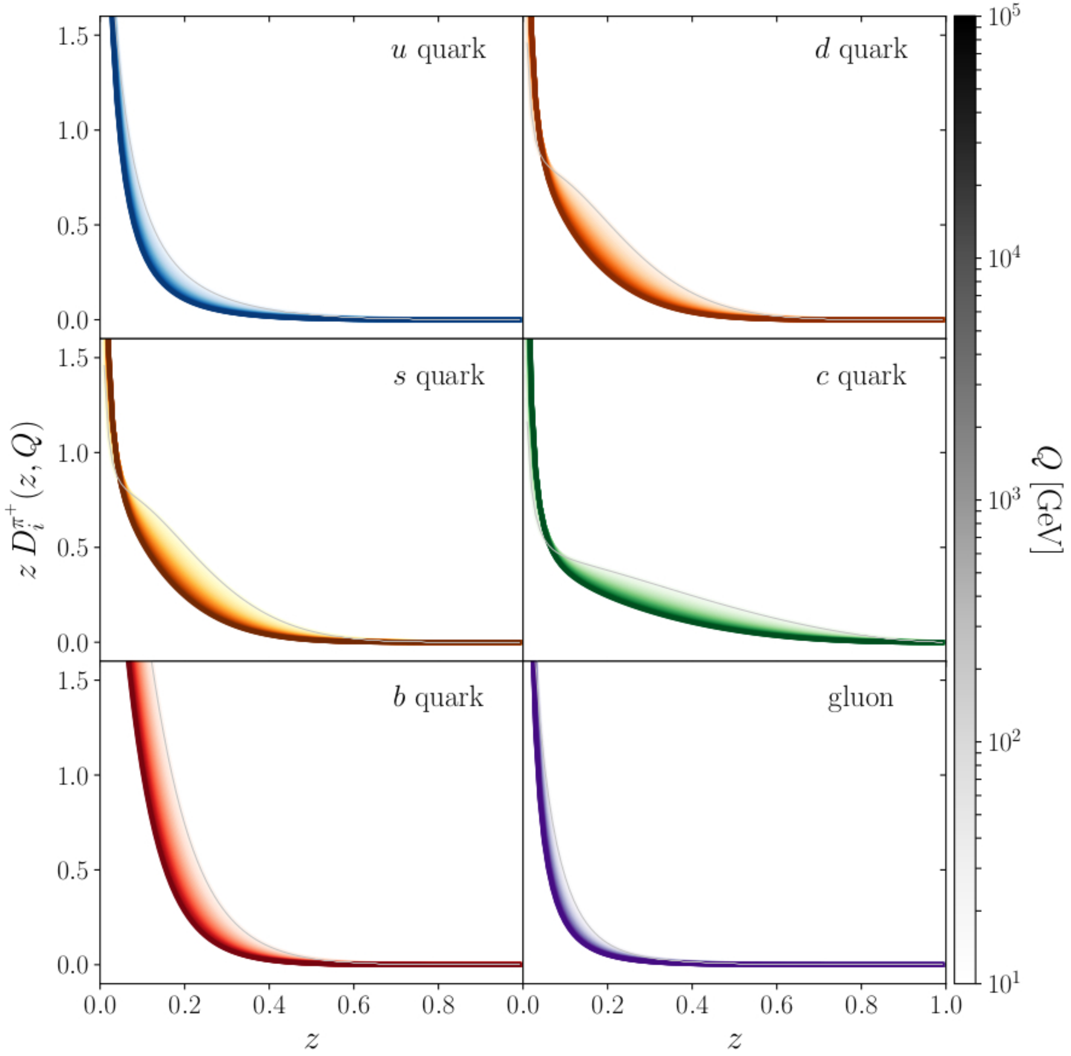

# Fit the non-extensive formula in $e^+e^-$ collisions

Parameter scale evolution

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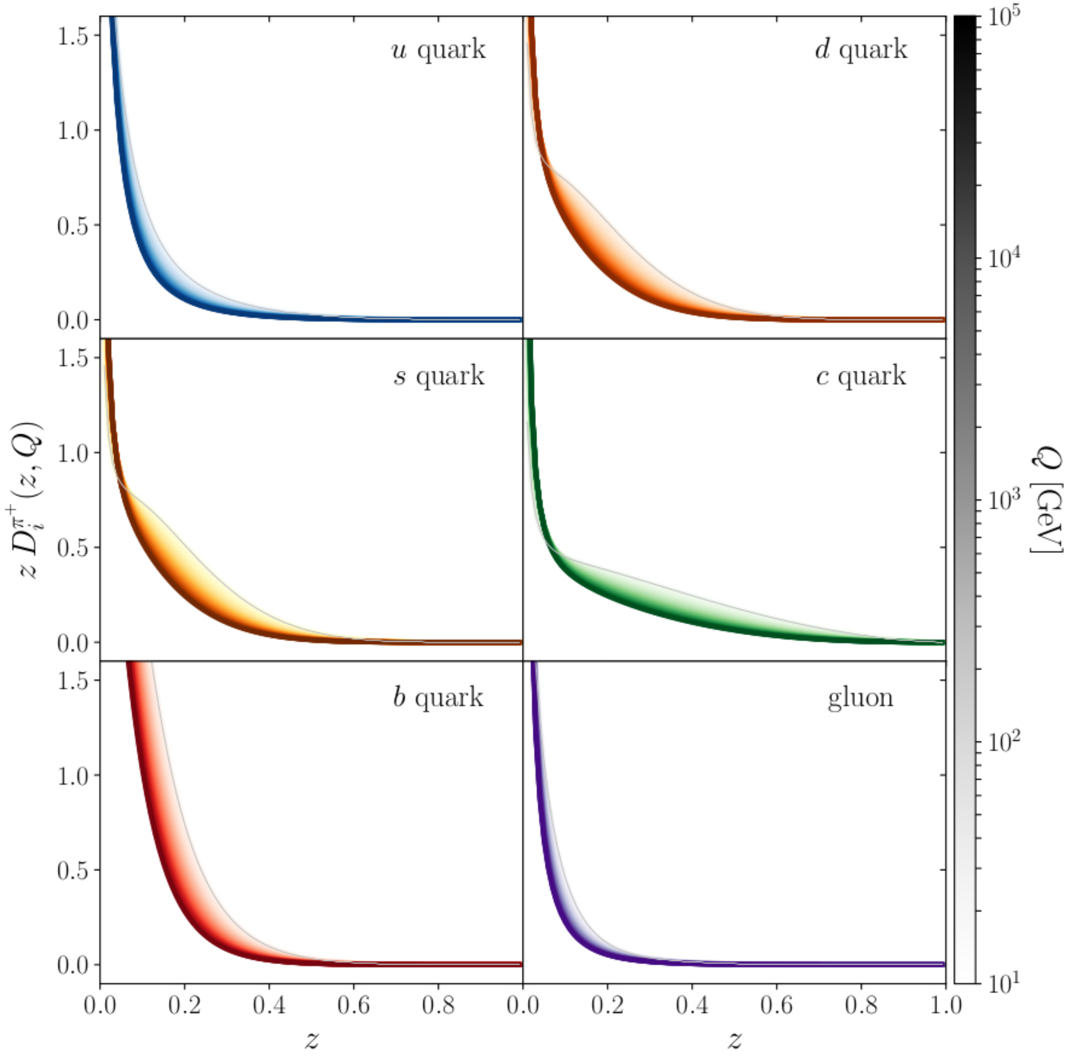
DGLAP is converted, fitted by a simply

formula for the *q*,*T*, & *N*



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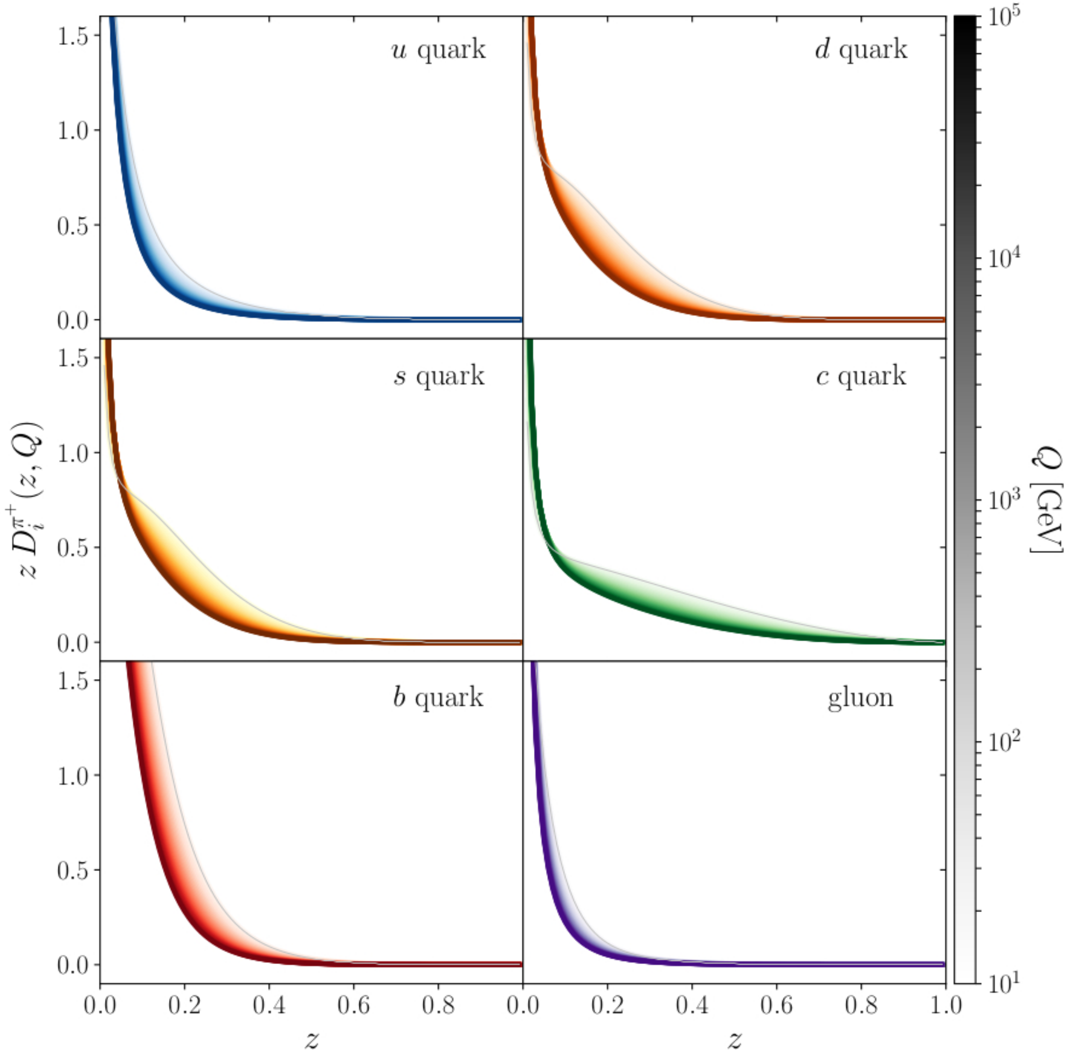
where

$$\bar{s} = \log\left[\frac{\log(Q^2/\Lambda^2)}{\log(Q_0^2/\Lambda^2)}\right]$$



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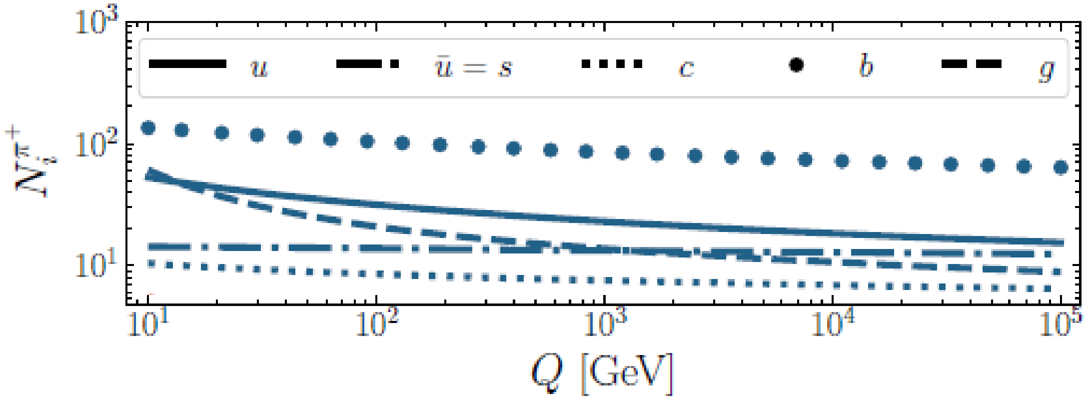


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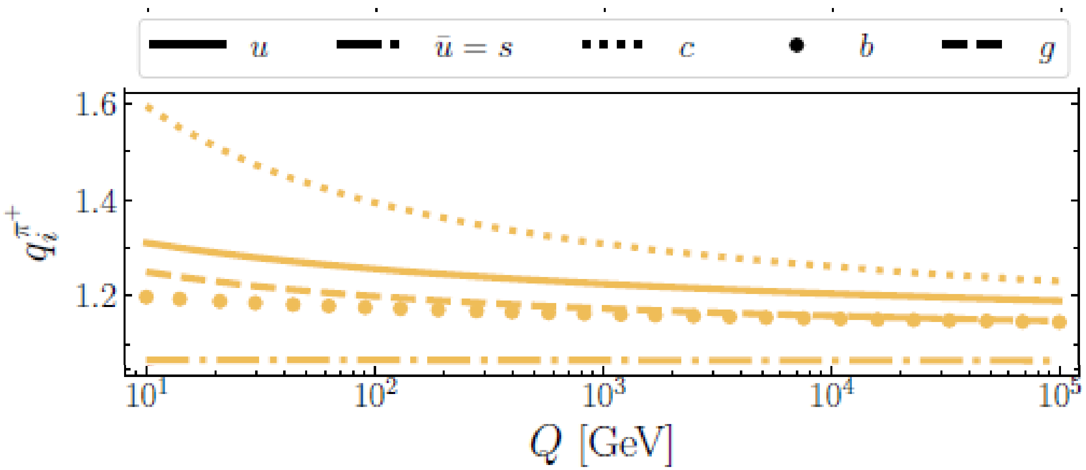

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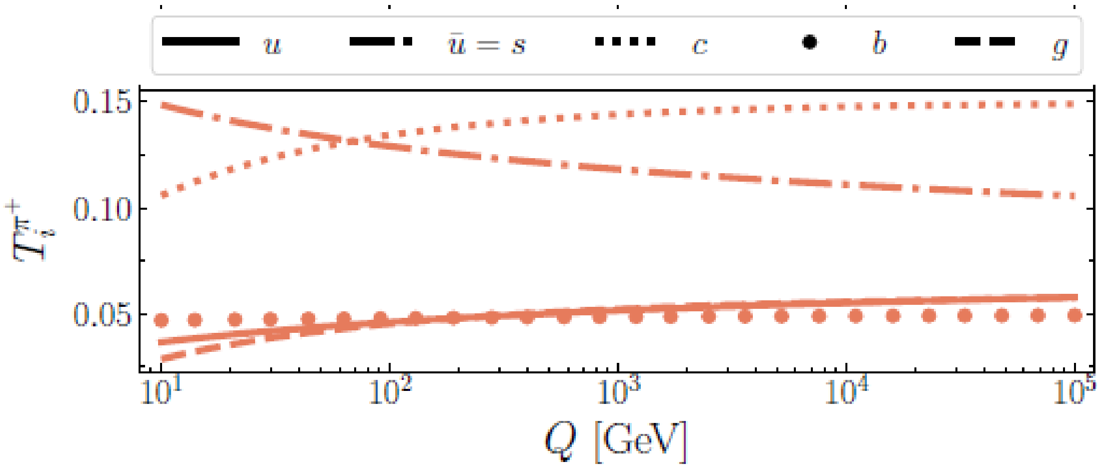

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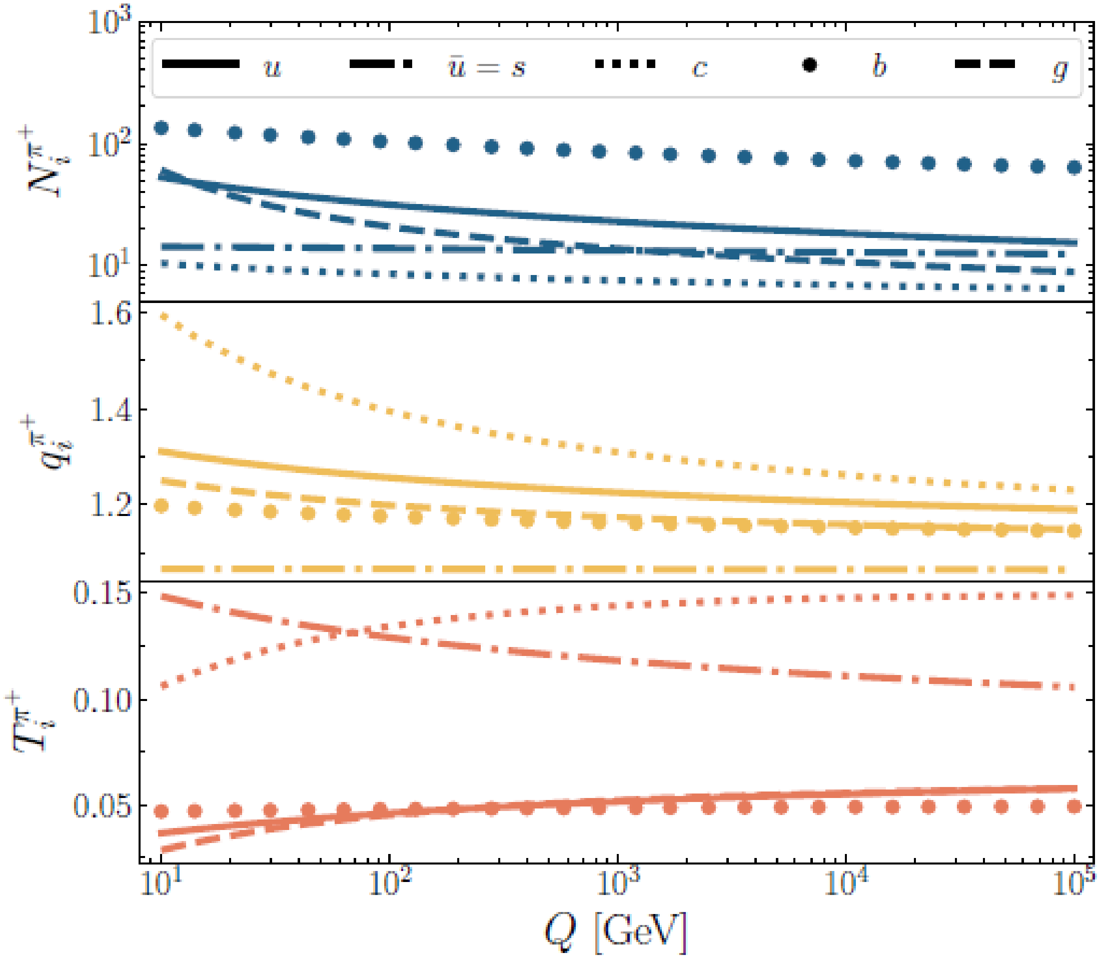


# Self-tests of the non-extensive formula in $e^+e^-$ collisions

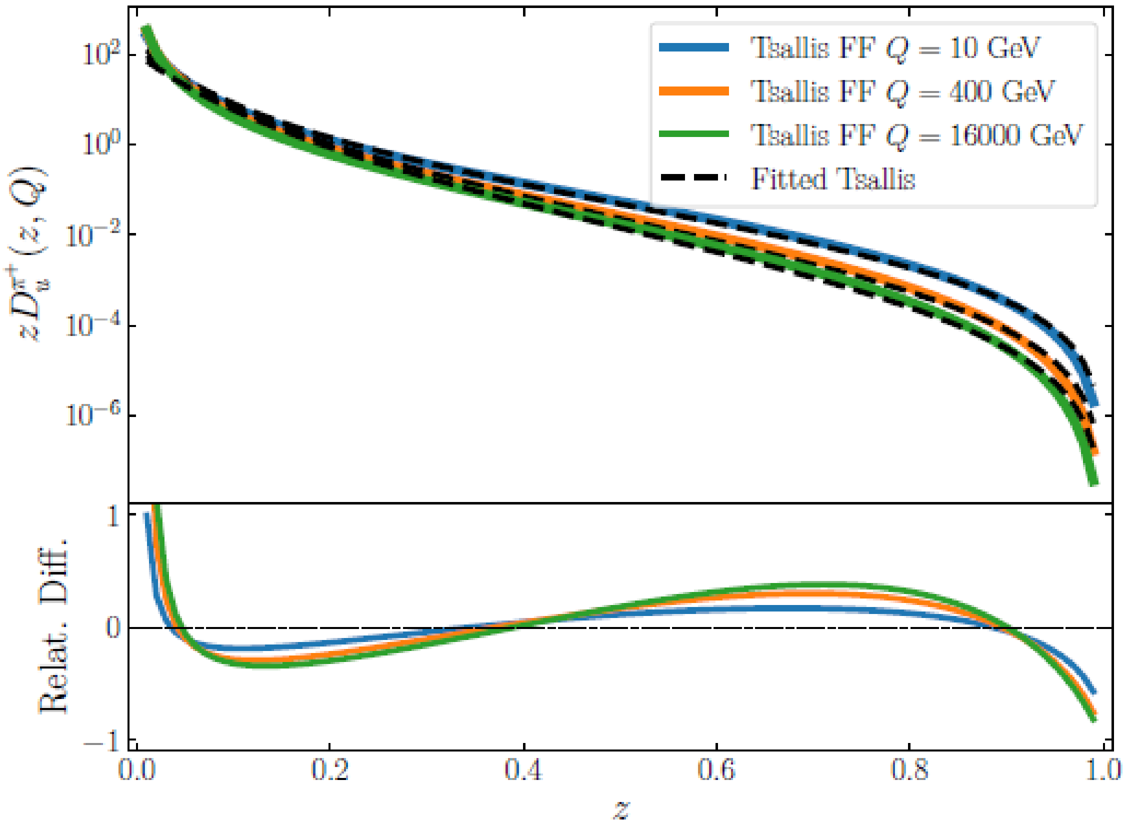
## Test of scale evolution

DGLAP evolution is given in LO:

$$\frac{dD_i^h(z,Q^2)}{d\log Q^2} = \sum_j \int_z^1 \frac{dx}{x} P_{ij}(z/x,Q^2) D_i^h(x,Q^2)$$

The real DGLAP scaling can be compared to the fit of the full, parametrized formula

$$D_i^h(z,Q) = N_i^h(Q)\left[1 - \frac{q_i^h(Q)-1}{T_i^h(Q)}\log(1-z)\right]^{-\frac{1}{q_i^h(Q)-1}}$$



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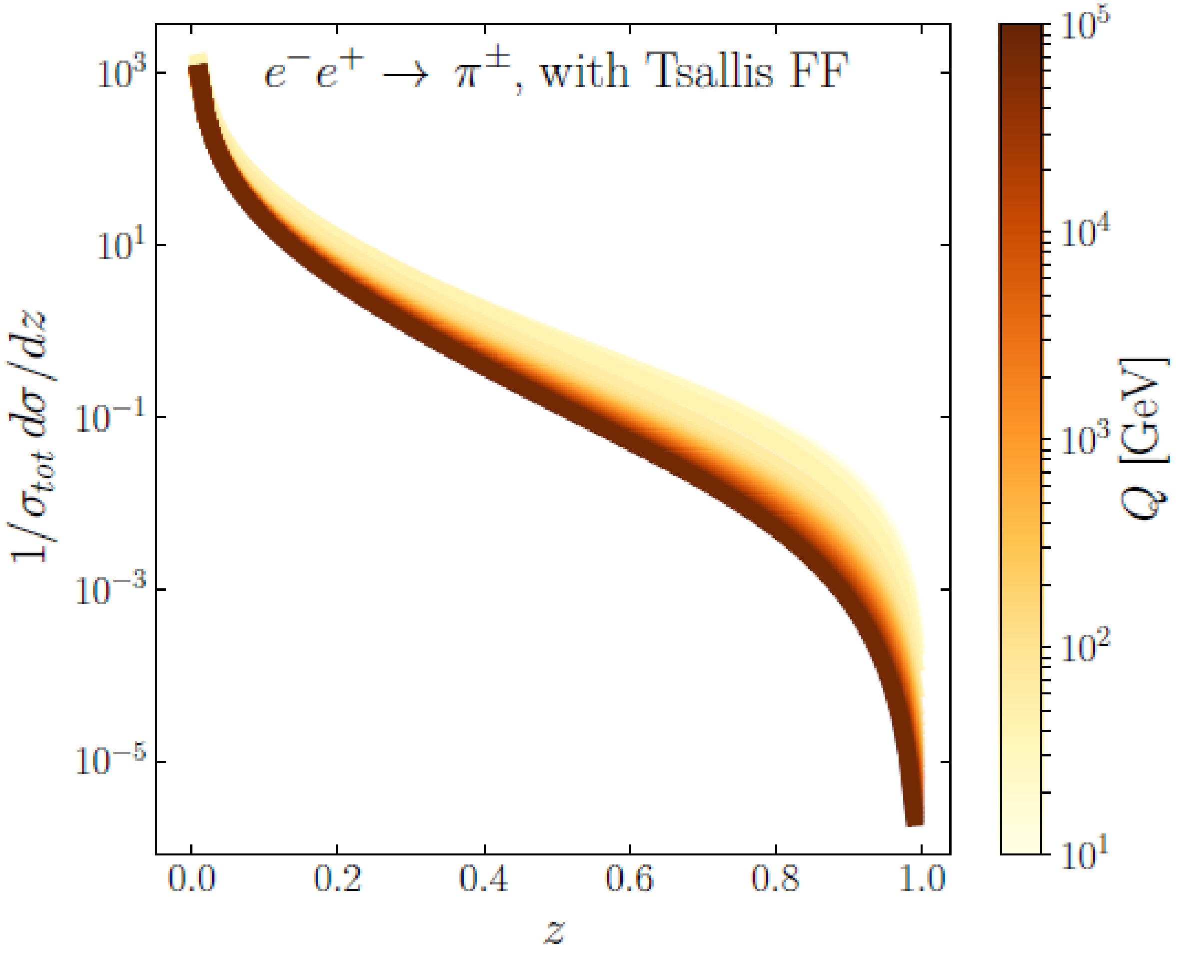



In full agreement with our earlier works'

scaling ansatz ~log(log(Q))

# Self-tests of the non-extensive formula in $e^+e^-$ collisions

## Channel contribution test

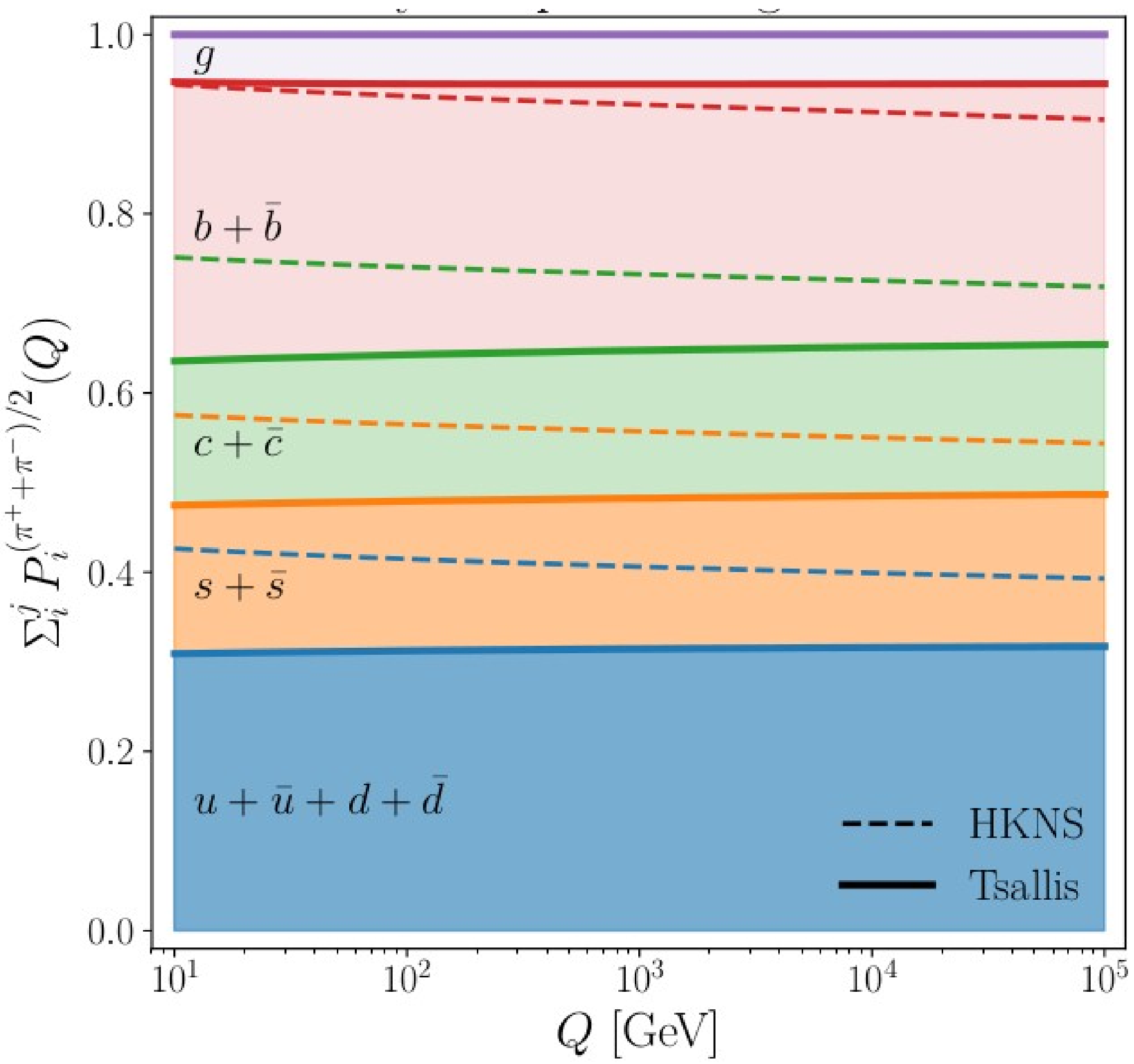
DGLAP evolution is given in LO:

$$\frac{dD_i^h(z,Q^2)}{d\log Q^2} = \sum_j \int_z^1 \frac{dx}{x} P_{ij}(z/x,Q^2) D_i^h(x,Q^2)$$

Using the sum rule,

$$1 = \sum_h \int_0^1 dz\, z D_i^h(z,Q)$$

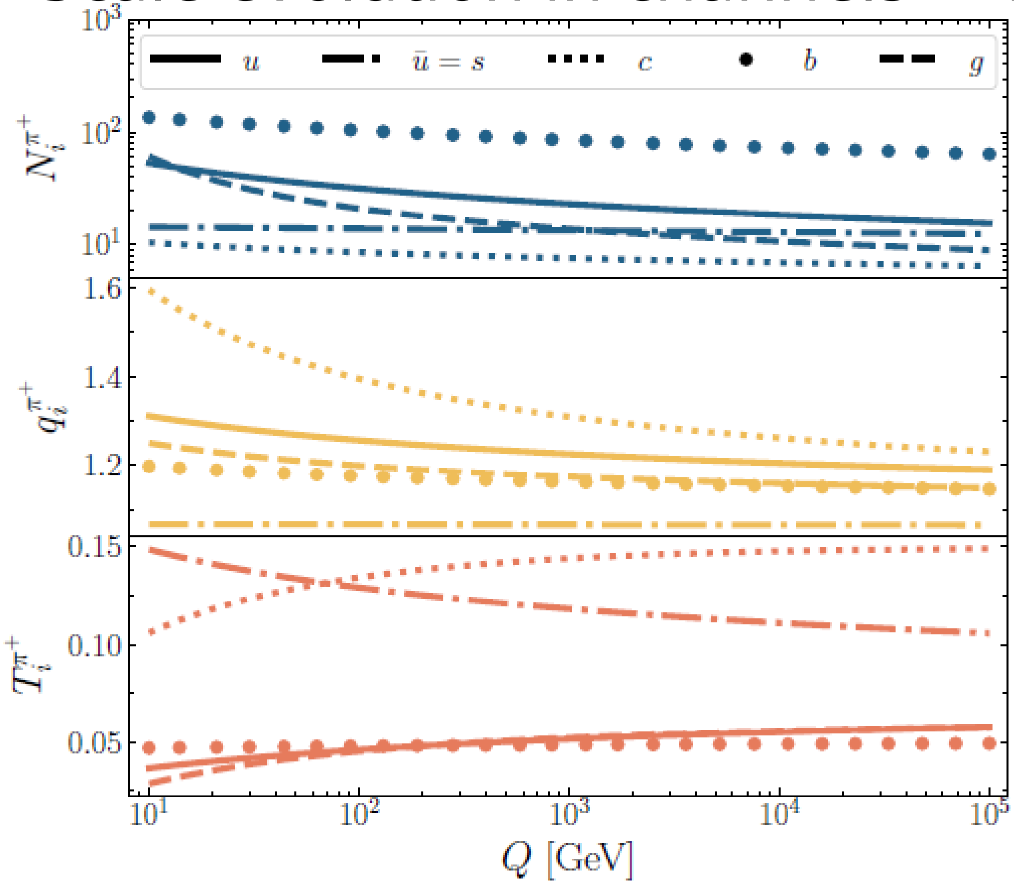
We calculated the evolution of the channels contribution (probability) to form charge-averaged pion.

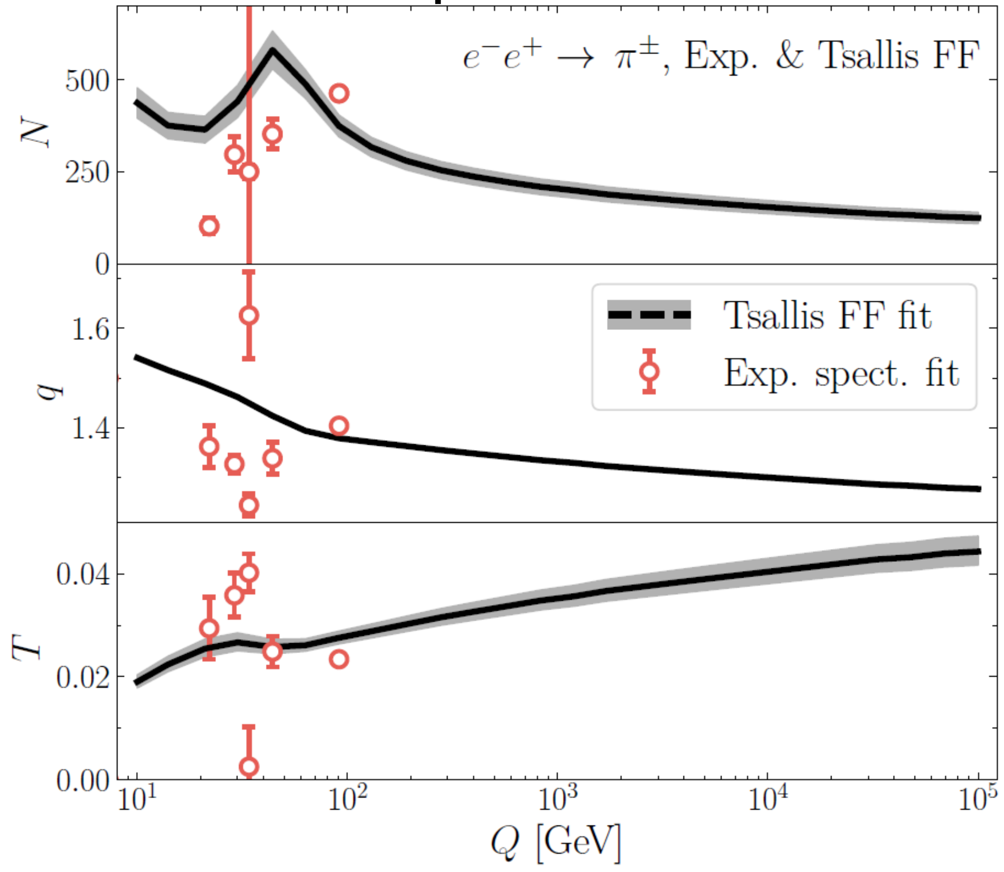


# Discussion & comparison to data

# Comparing non-extensive FF with $e^+e^-$ data

Scale evolution in channels → full formula + errors + pion data



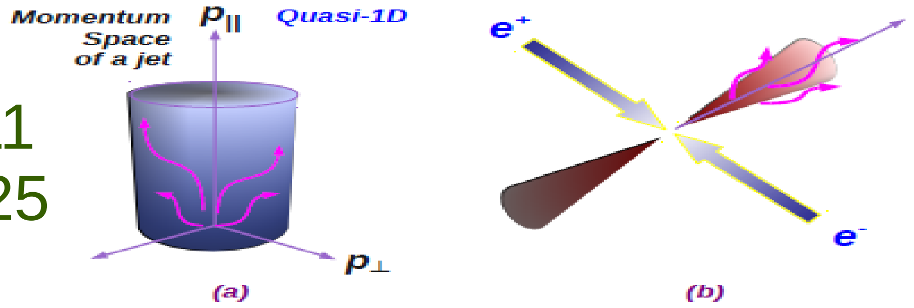
# Comparing non-extensive FF with $e^+e^-$ data

## Comparison to earlier, global pion data fits

Parameters from channels (color)

| $i \to \pi^+$ | $q$ | $1/T$ | $N$ |
|---|---|---|---|
| $u$ | 1.455 | 62.72 | 207.1 |
| $\bar{u} = s$ | 1.063 | 4.850 | 13.68 |
| $c$ | 2.238 | 25.89 | 14.97 |
| $b$ | 1.211 | 21.45 | 151.6 |
| $g$ | 1.059 | 17.05 | 53.81 |

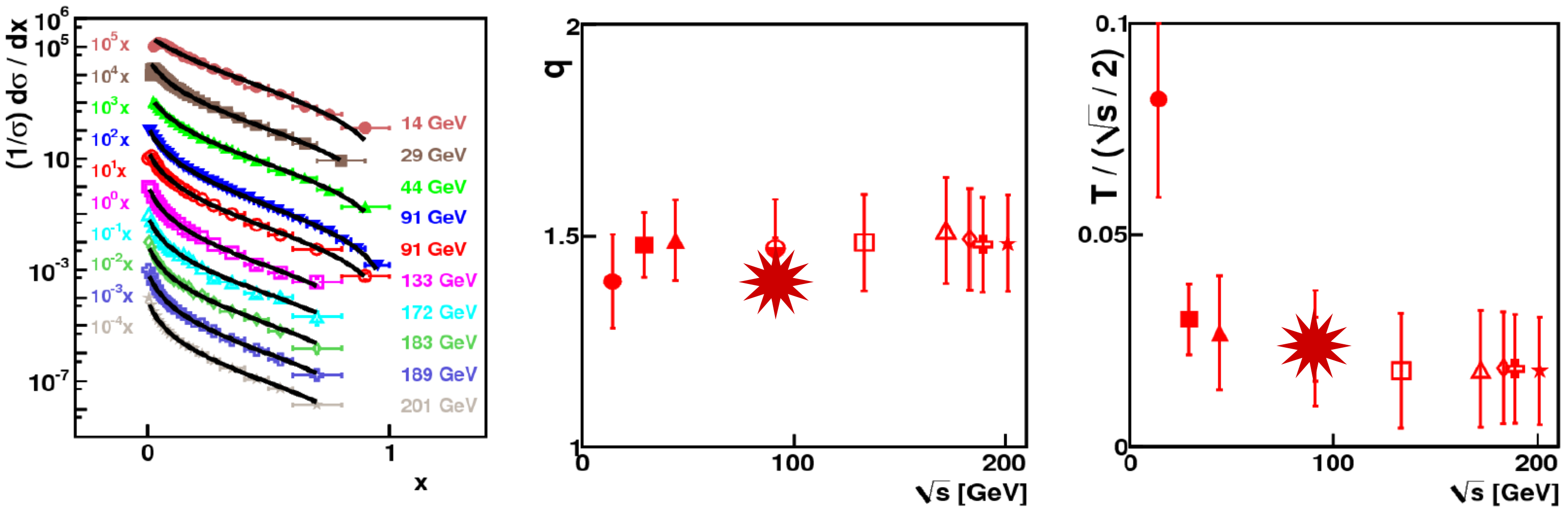
PLB701 (2011) 111
PLB718 (2012) 125




Overall parameters (black)

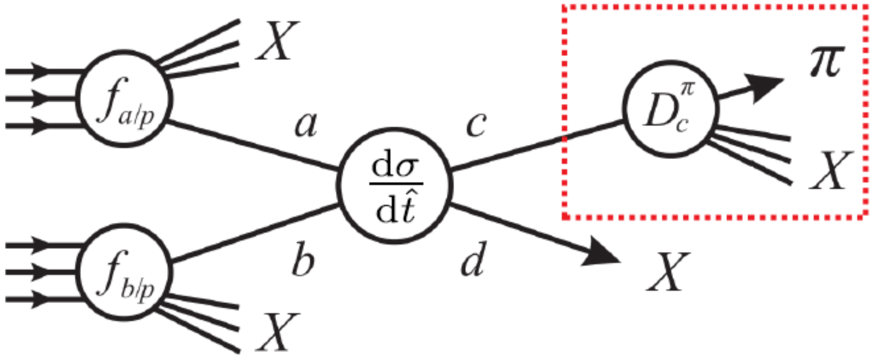
Non-extensivity: $q = 1.39$

T parameter: $1/T = 40$



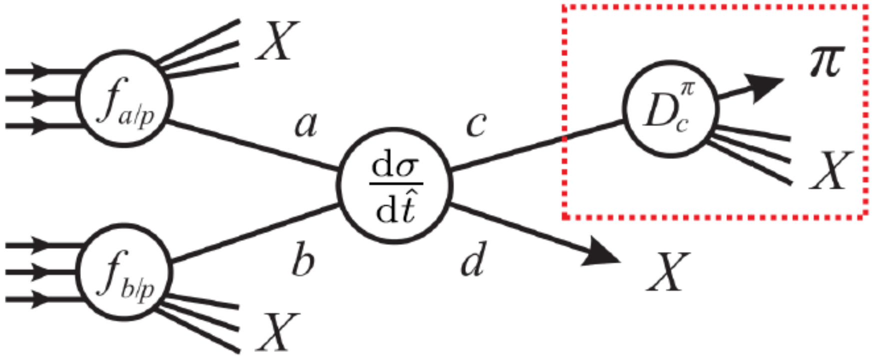
# Comparing non-extensive FF with pp data

Test of non-extensive FFs within the kTpQCD_v20 model



# Comparing non-extensive FF with pp data

Test of non-extensive FFs within the kTpQCD_v20 model




$$D_i^h(z,Q) = N_i^h(Q)\left[1 - \frac{q_i^h(Q)-1}{T_i^h(Q)}\log(1-z)\right]^{-\frac{1}{q_i^h(Q)-1}}$$

$$N_i^h = a_{N_i^h} + b_{N_i^h}\,\bar{s} + c_{N_i^h}\,\bar{s}^2 + d_{N_i^h}\,\bar{s}^3,$$

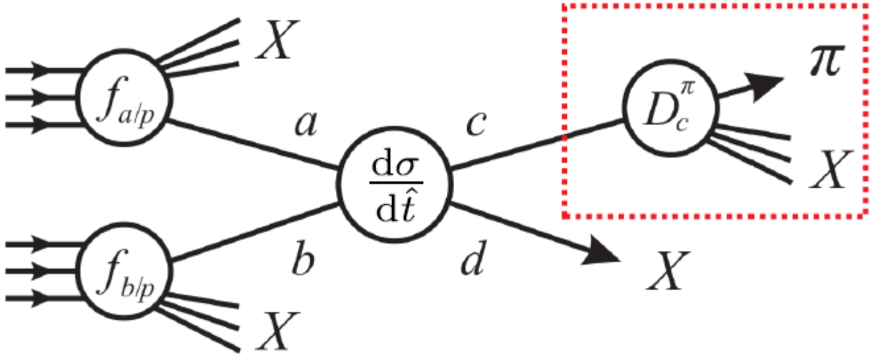
$$q_i^h = a_{q_i^h} + b_{q_i^h}\,\bar{s} + c_{q_i^h}\,\bar{s}^2 + d_{q_i^h}\,\bar{s}^3,$$

$$T_i^h = a_{T_i^h} + b_{T_i^h}\,\bar{s} + c_{T_i^h}\,\bar{s}^2 + d_{T_i^h}\,\bar{s}^3,$$

$$\bar{s} = \log\left[\frac{\log(Q^2/\Lambda^2)}{\log(Q_0^2/\Lambda^2)}\right] \qquad Q_0 = \begin{cases} 1\text{ GeV}, & \text{for } u,d,s,g \\ m_c & \text{for } c \\ m_b & \text{for } b \end{cases}$$

# Comparing non-extensive FF with pp data

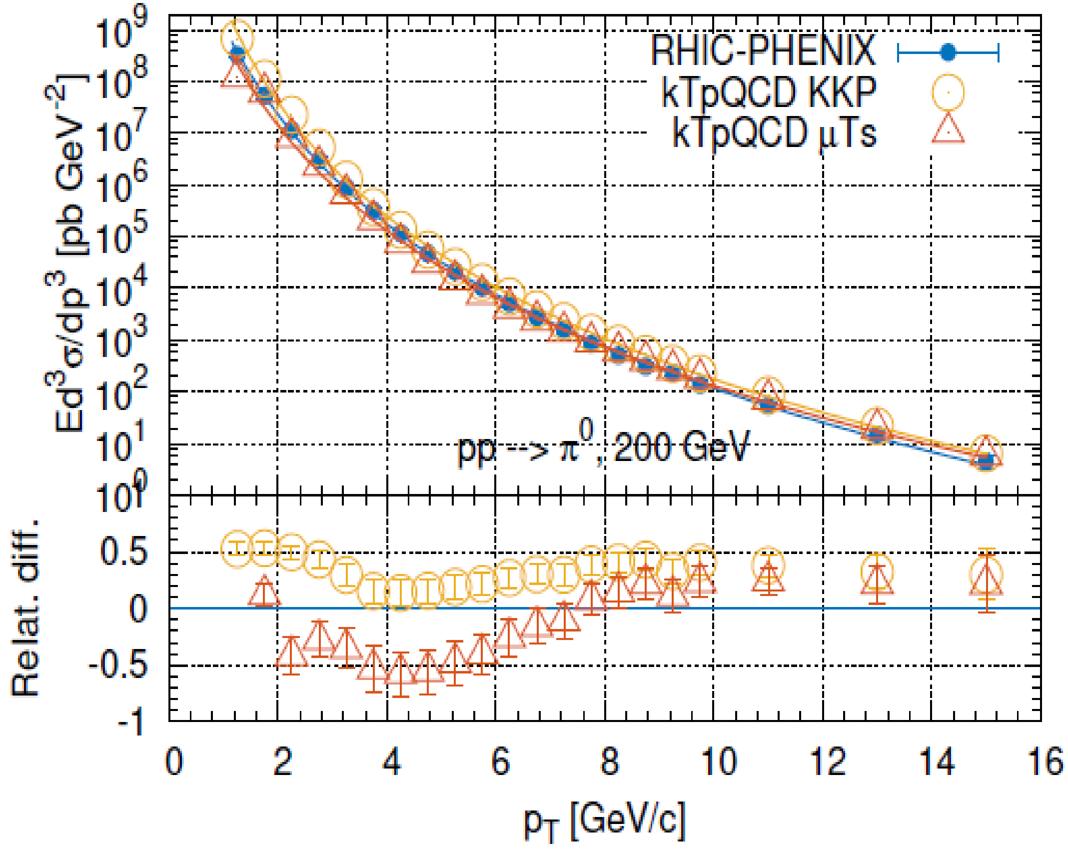
Test of non-extensive FFs within the kTpQCD_v20 model




$$D_i^h(z,Q) = N_i^h(Q)\left[1 - \frac{q_i^h(Q)-1}{T_i^h(Q)}\log(1-z)\right]^{-\frac{1}{q_i^h(Q)-1}}$$

$$N_i^h = a_{N_i^h} + b_{N_i^h}\,\bar{s} + c_{N_i^h}\,\bar{s}^2 + d_{N_i^h}\,\bar{s}^3,$$
$$q_i^h = a_{q_i^h} + b_{q_i^h}\,\bar{s} + c_{q_i^h}\,\bar{s}^2 + d_{q_i^h}\,\bar{s}^3,$$
$$T_i^h = a_{T_i^h} + b_{T_i^h}\,\bar{s} + c_{T_i^h}\,\bar{s}^2 + d_{T_i^h}\,\bar{s}^3,$$

$$\bar{s} = \log\left[\frac{\log(Q^2/\Lambda^2)}{\log(Q_0^2/\Lambda^2)}\right] \qquad Q_0 = \begin{cases} 1\ \mathrm{GeV}, & \text{for } u,d,s,g \\ m_c & \text{for } c \\ m_b & \text{for } b \end{cases}$$



# Comparing non-extensive FF with $e^+e^-$ data

'Tsallis thermometer' → full formula + errors + pion data

Parameters from KKP, HKNS? And Tsallis-like Ffs: channels (color), global (black)

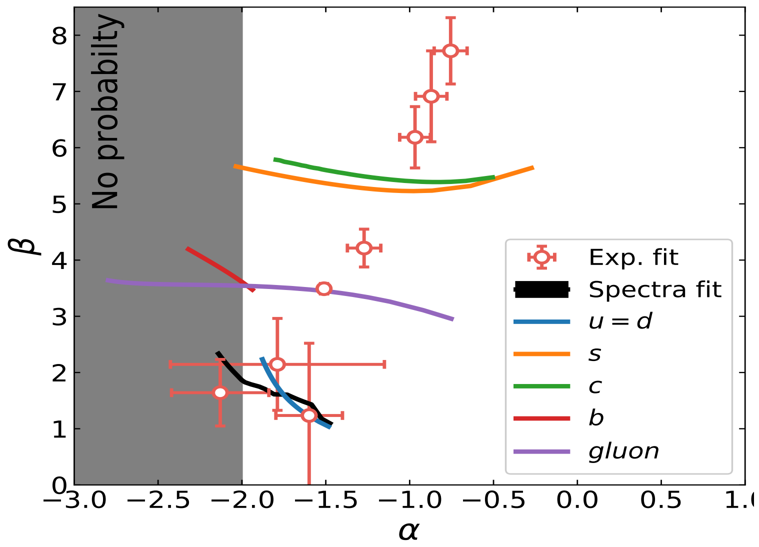
$$D_i^h(z,Q^2) = N_i^h z^{\alpha_i^h}(1-z)^{\beta_i^h}$$

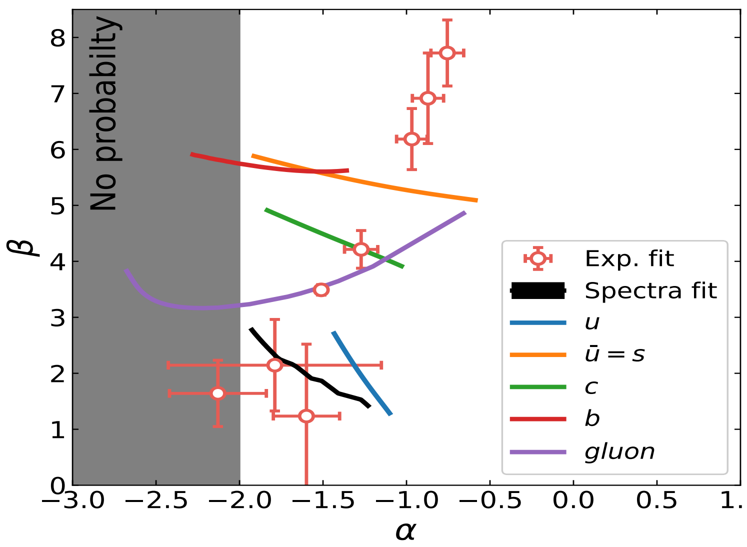
$$D_i^h(z,Q) = N_i^h(1-z)\left[1-\frac{q_i^h-1}{T_i^h}\log(1-z)\right]^{-\frac{1}{q_i^h-1}}$$

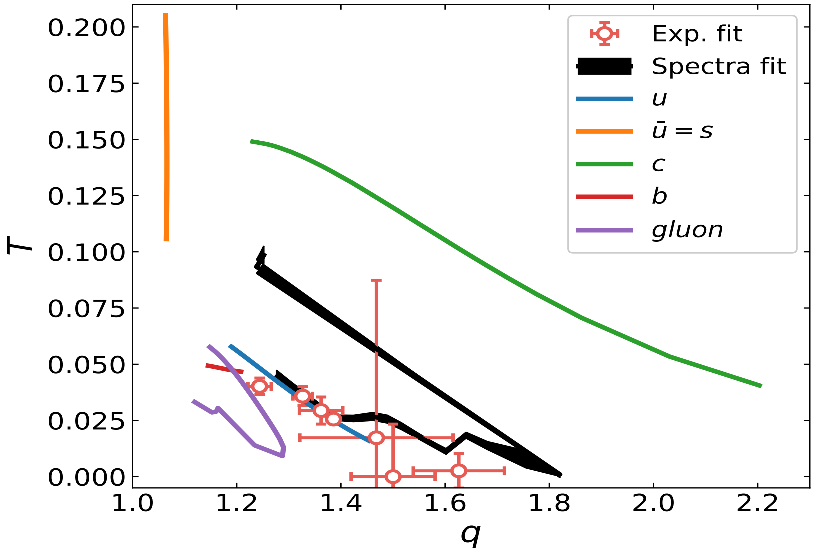
KKP fit

HKNS fit

Tsallis-like fit



# Summary

Aim: non-extensive fragmentation function parametrization

→ Pion FFs are available for tests, fit on one dataset so far

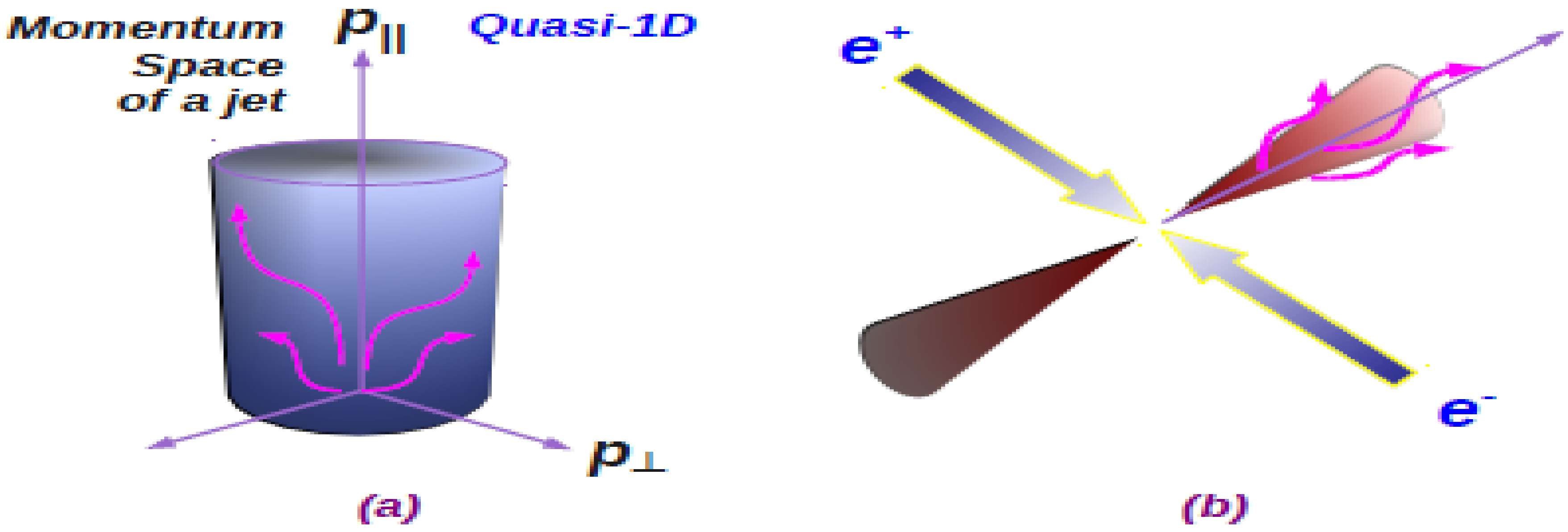
So far we have:

- Non-extensive phenomena motivated Tsallis-like distribution with physical meaning of the FF parameters
- Scale evolution is fully observed in (*q,T,N*) for channels & overall
  - Other models: in comparison to HKNS, AKK present similar trends
  - Data: comparisons & tests with other pion data fits well
  - Better low-z behavior: even applying in pp spectra.
  - Model: similarities with jet (1D) themodynamics & multiplicities

Next: resolve uncertainties, an fird the real minimalizaion

# BACKUP

# Comparing non-extensive FF with $e^+e^-$ data




K. Ürmössy, G.G. Barnaföldi, T.S. Bíró:

- Microcanonical Jet-Fragmentation in pp at LHC energies:
  Phys. Lett. B701 (2011) 111
- Generalized Tsallis distribution in $e^+e^-$ collisons
  Phys. Lett. B718 (2012) 125

# Comparing non-extensive FF with $e^+e^-$ data

Tsallis thermodynamics

$$\frac{1}{\sigma}\frac{d\sigma}{dx} = A x^{D-1}\left(1 + \frac{q-1}{T/(\sqrt{s}/2)}x\right)^{-1/(q-1)}$$

$$q = 1 + 1/(\alpha + D + 1)$$

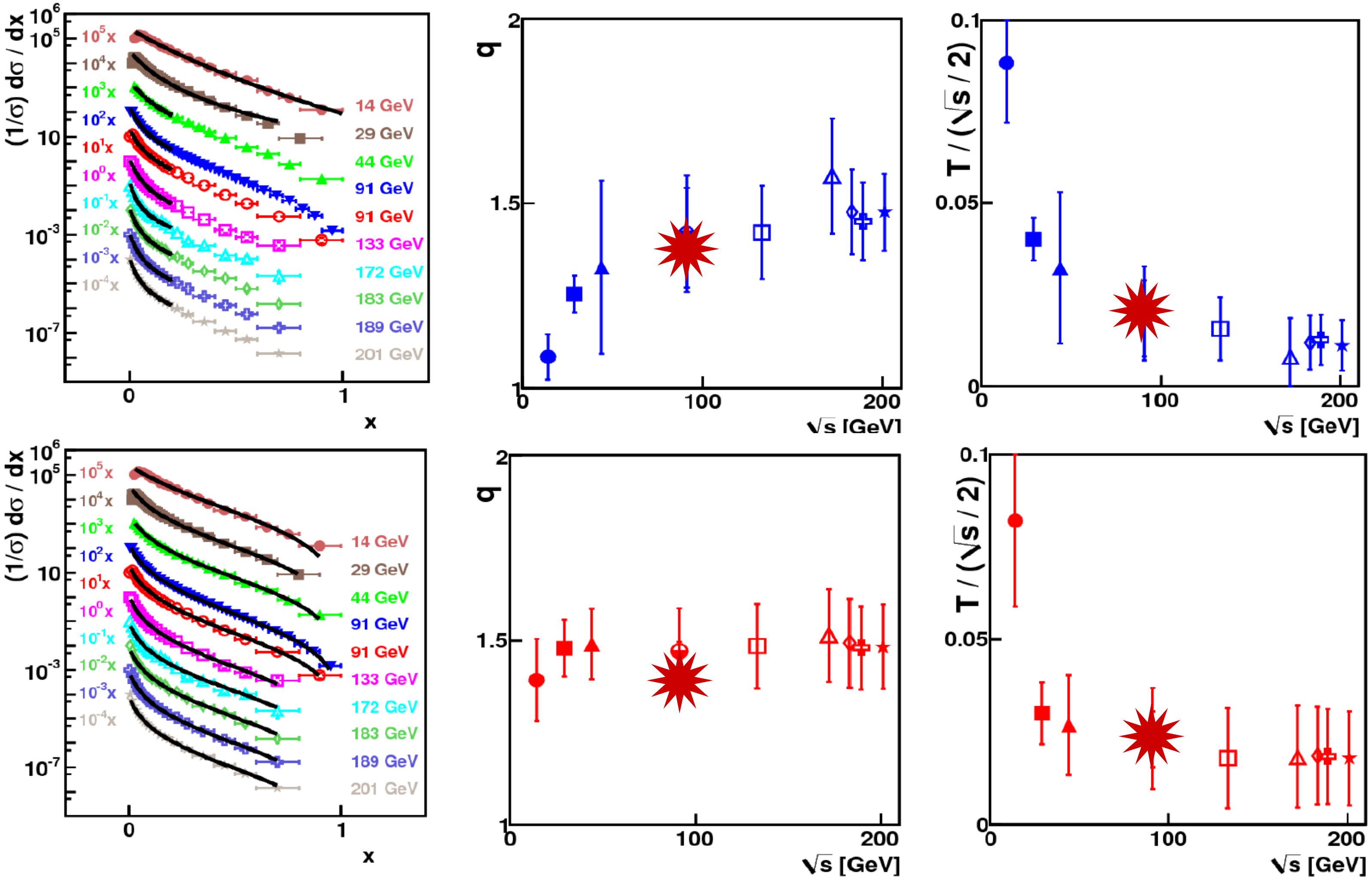
$$T = (\sqrt{s}/2)\beta/(D(\alpha + D + 1))$$

Microcanonical Tsallis

$$\frac{1}{\sigma}\frac{d\sigma}{dx} = \frac{A x^{D-1}(1-x)}{\left(1 - \frac{q-1}{T/(\sqrt{s}/2)}\ln(1-x)\right)^{1/(q-1)}}$$

$$q = 1 + 1/(\alpha + D + 1)$$

$$T = (\sqrt{s}/2)\beta/(D(\alpha + D + 1))$$



# Comparing non-extensive FF with $e^+e^-$ data

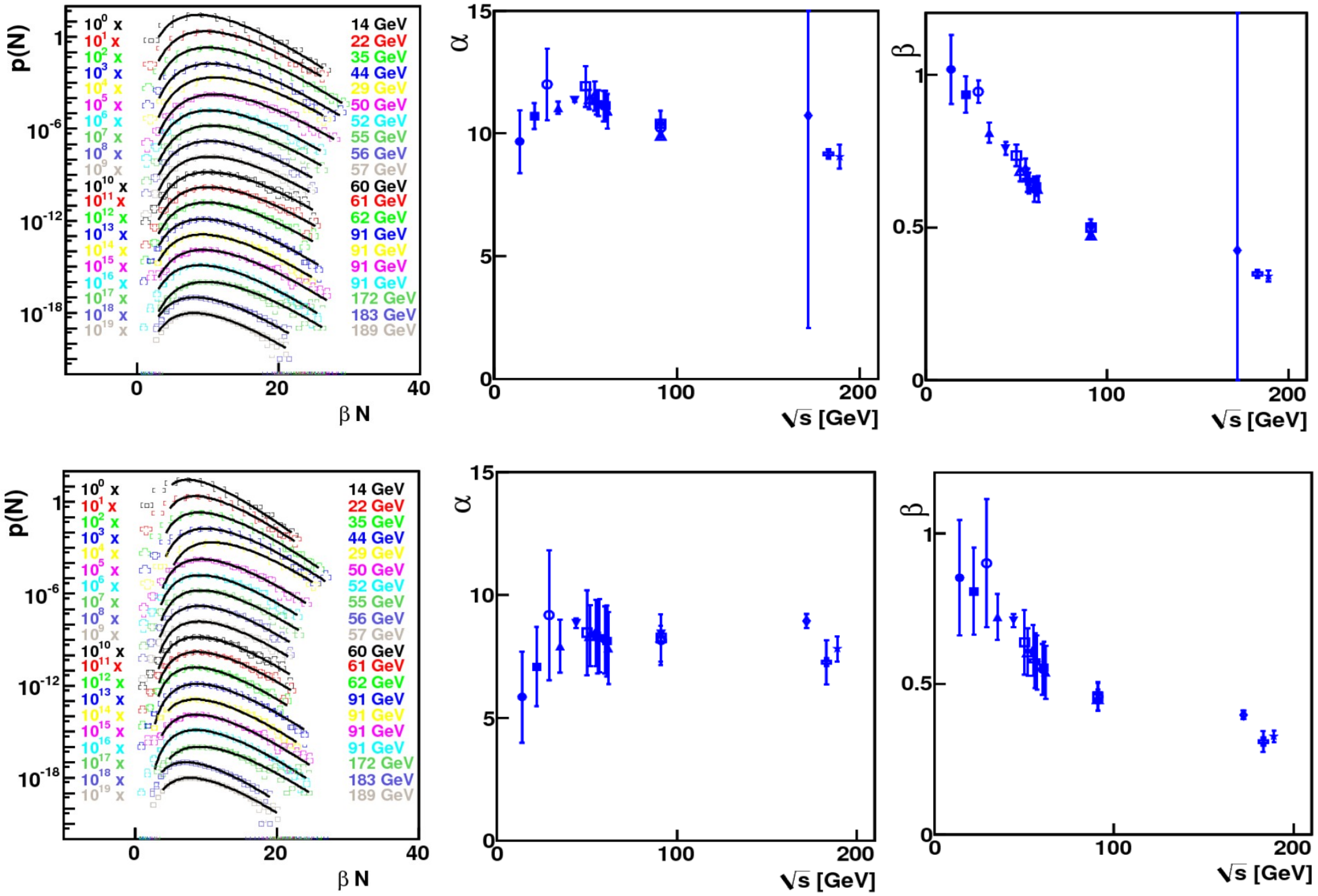
Tsallis thermodynamics

$$p(N) = A_m \, N^{\alpha-1} e^{-\beta N}$$

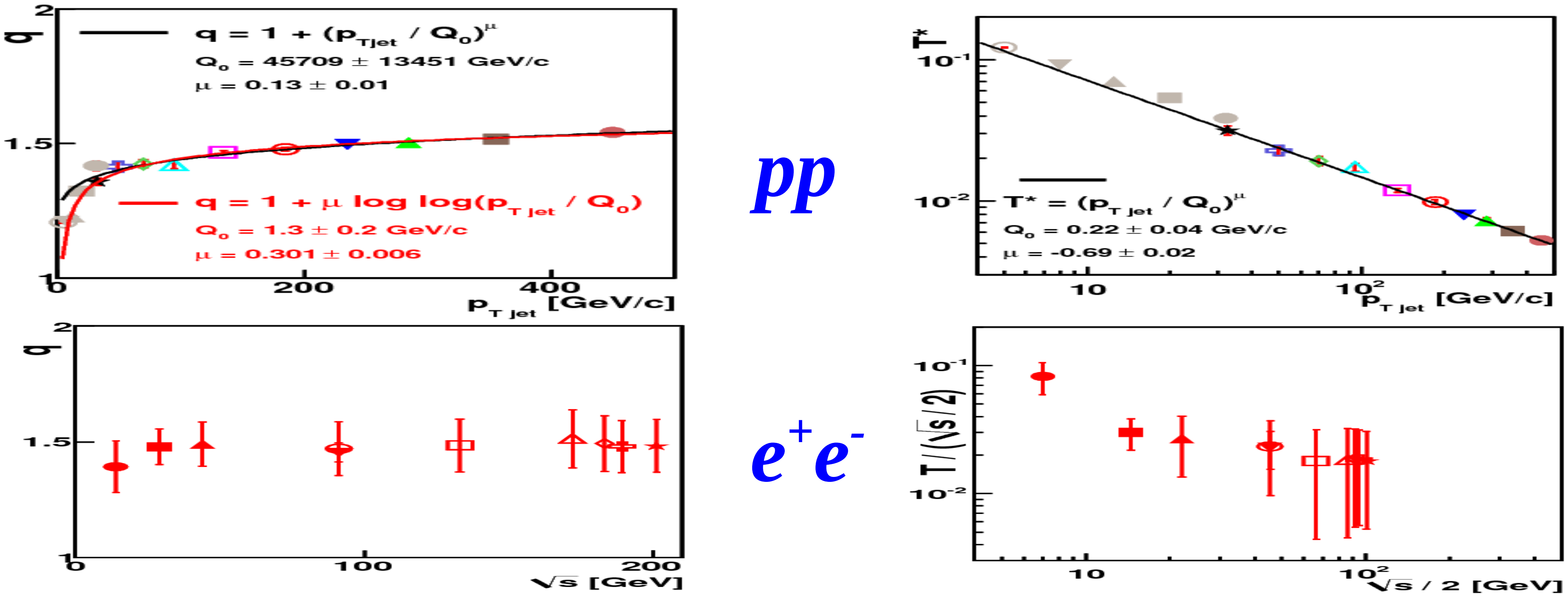
Microc

$$p(N) = A_m \, (N - N_0)^{\alpha-1} e^{-\beta (N-N_0)}$$

$$N_0 = 1 + 2/D$$



# Comparing non-extensive FF with $e^+e^-$ & *pp* data




K Ürmössy, GGB, TS Biró,
PLB 710 (2011) 111, PLB 718 (2012) 125.

- Energy dependence (hard)
  - Parameters *q* seem to increase & saturate at high energies
  - Parameter *T* is decreasing & saturate with increasing energy

# The non-extensive statistical approach

**Hadron spectra in *pp* collisions can be described by the *Tsallis distribution*:**

$$\frac{dN}{d^3\mathbf{p}} \propto \left[1 + \frac{q-1}{T}(m_T - m)\right]^{-1/(q-1)} .$$

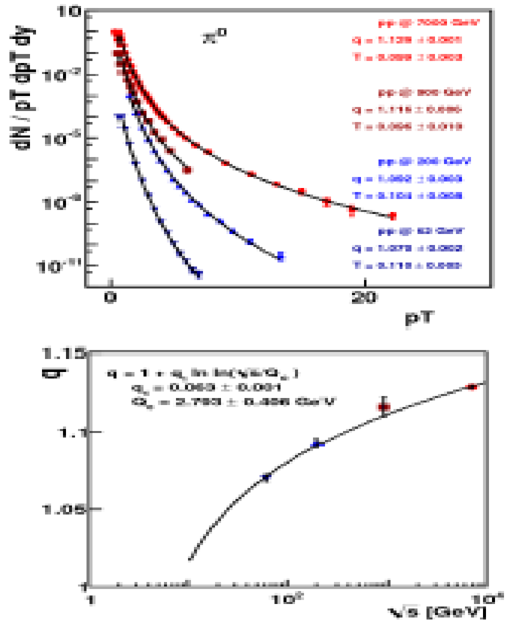
**$\pi$ spectra in *pp* collisions depends *similarly on* $\sqrt{s}$ and on the multiplicity *N***

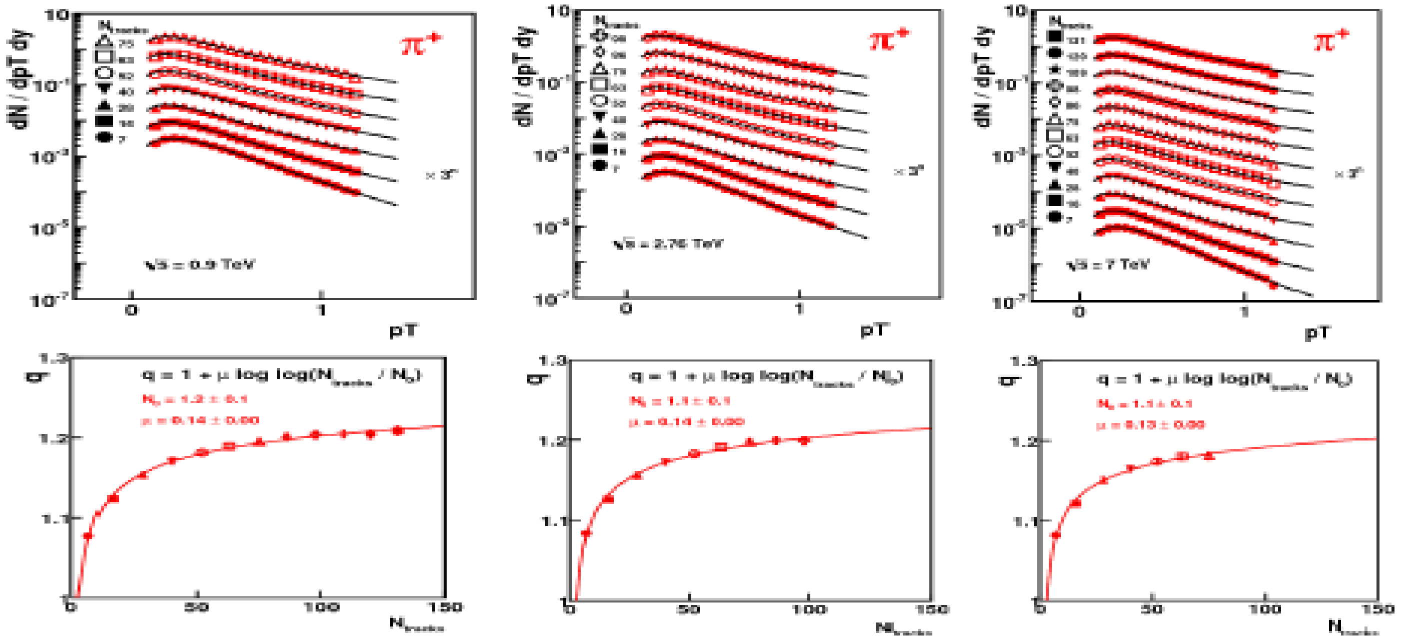
$$q(s) = 1 + q_1 \ln\ln(\sqrt{s}/Q_0) ,$$
$$q(N) = 1 + \mu \ln\ln(N/N_0) .$$

√s = fix

N = fix

arXiv:1405.3963, 1501.02352, 1501.05959



# The non-extensive statistical approach

**Hadron spectra in *pp* collisions can be described by the *Tsallis distribution*:**

$$\frac{dN}{d^3\mathbf{p}} \propto \left[1 + \frac{q-1}{T}(m_T - m)\right]^{-1/(q-1)} .$$

**π spectra in *pp* collisions depends *similarly on √s* and on the multiplicity**

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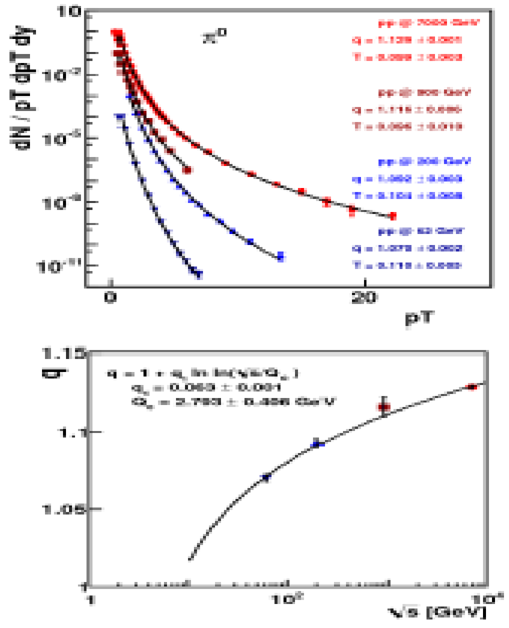
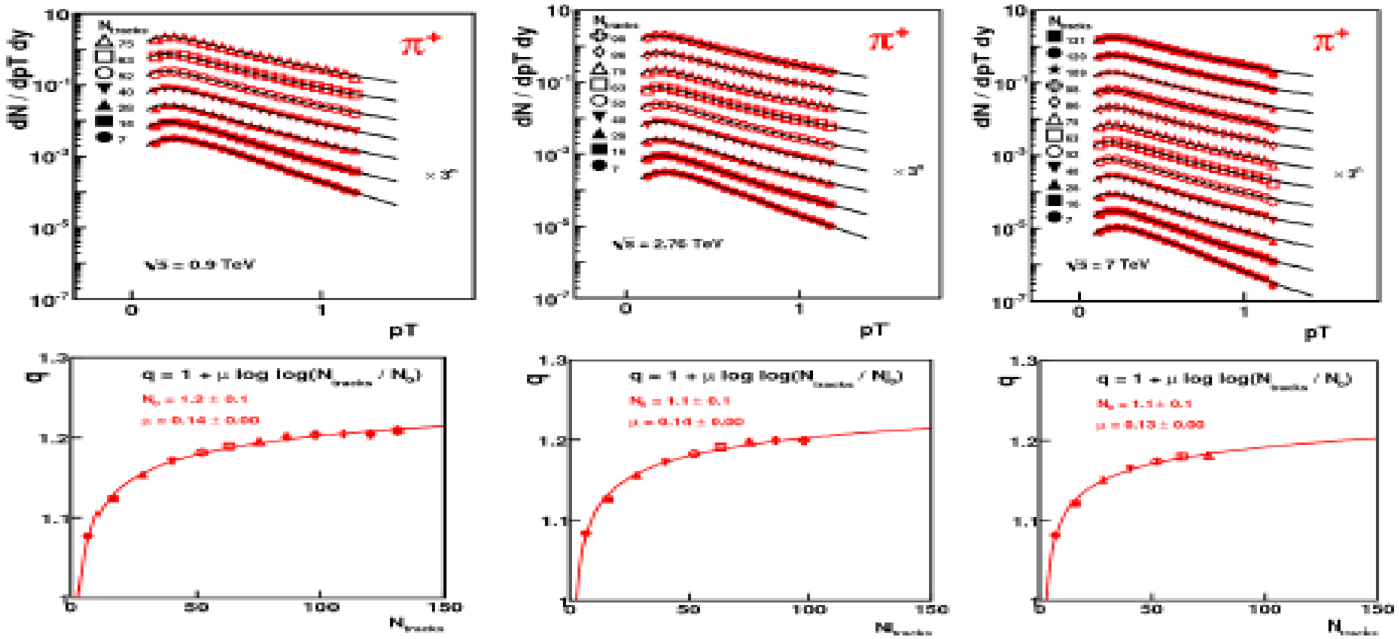
**√s = fix**




**N = fix**

arXiv:1405.3963, 1501.02352, 1501.05959



# In pp: the Tsallis thermometer on the *T-(q-1)* plane

- Parameter space

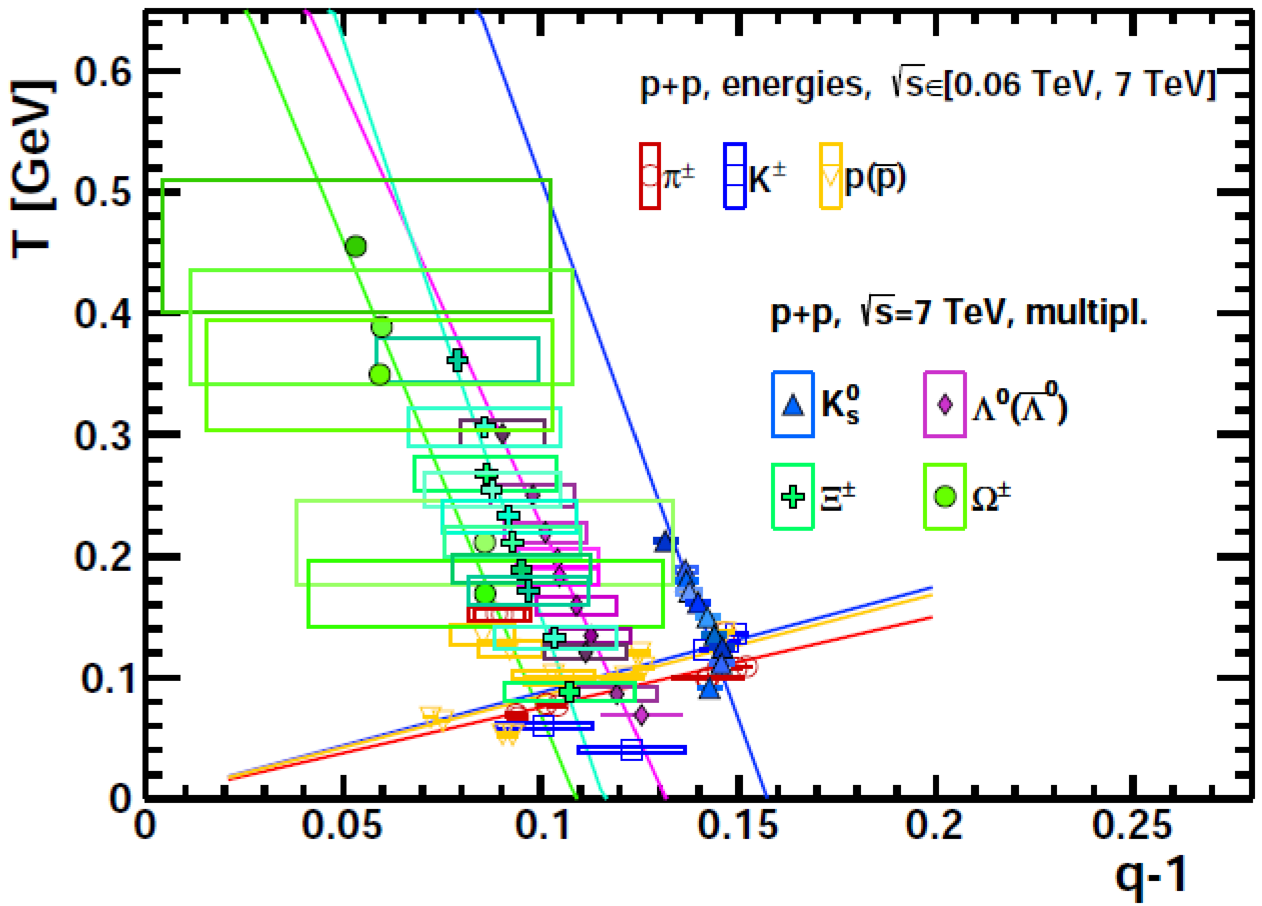
  (i) c.m. energy makes changes along q-axis

  (ii) multiplicity vary the parameter T

  (iii) the mass hierarchy can be seen clearly with bunches

  (iv) valid for pp

  (v) T & q are connected



# In pA: the Tsallis thermometer on the *T-(q-1)* plane

- Parameter space

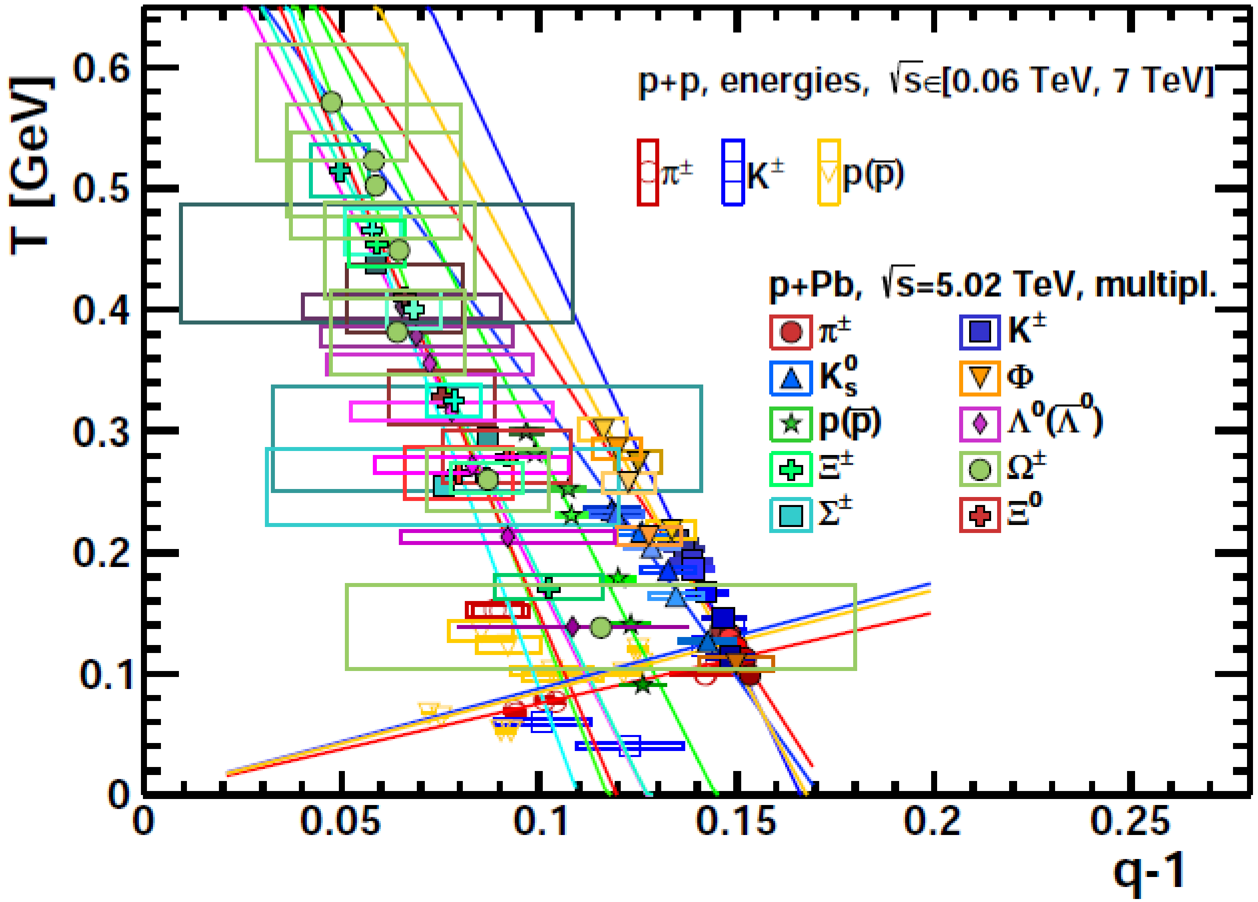
  (i) c.m. energy makes changes along q-axis

  (ii) multiplicity vary the parameter T

  (iii) the mass hierarchy can be seen clearly with bunches

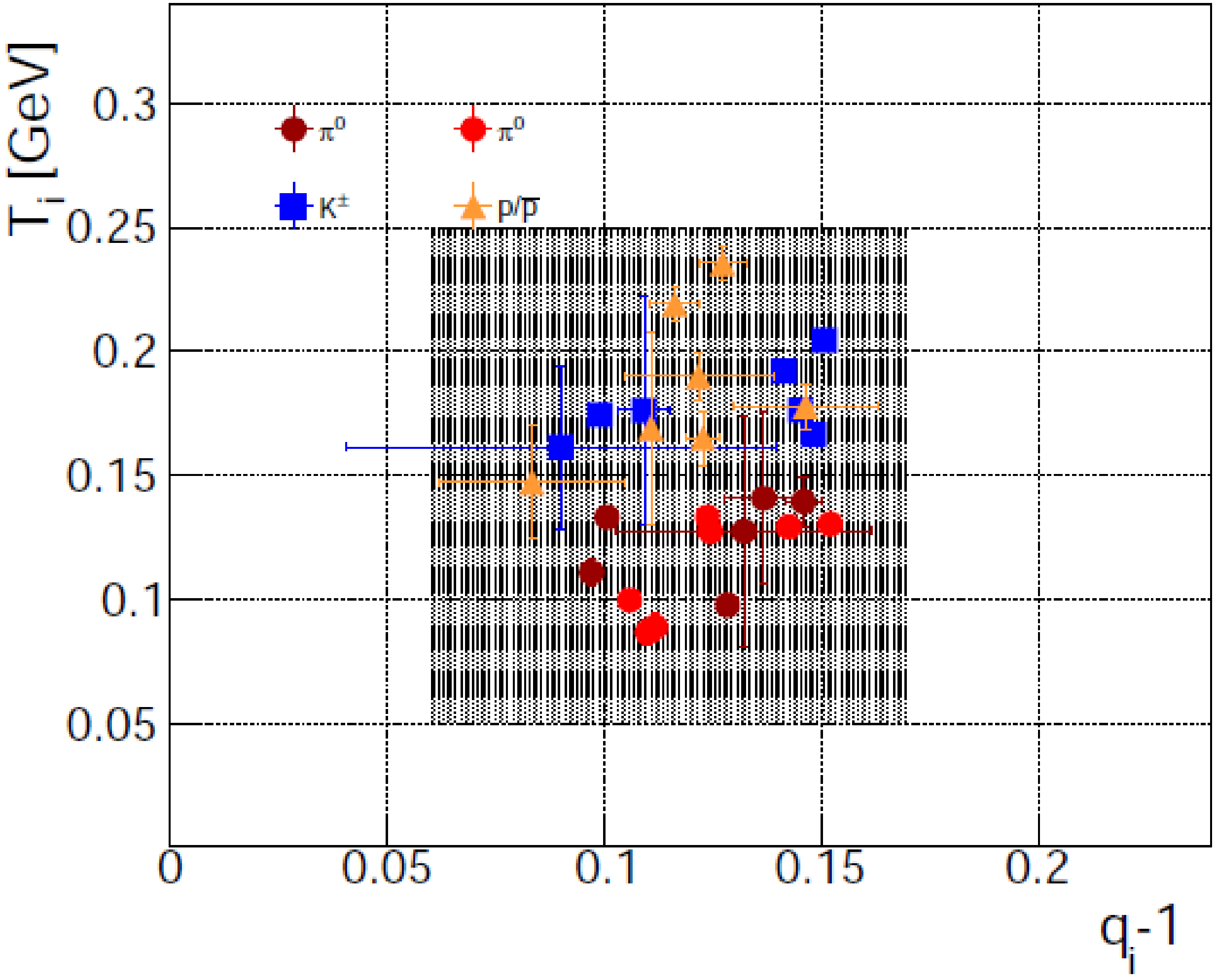
  (iv) valid for pp & pA

  (v) T & q are connected



# In pp: the Tsallis thermometer on the *T-(q-1)* plane

- Measurements in pp
  - – parameter space is compact, especially in q
  - – c.m. energy makes changes along q-axis
  - – T & q are connected



# In pp: the Tsallis thermometer on the *T-(q-1)* plane

- Theory in pp

  Both are overlapping

  – PYTHIA8 deviates where no statistics in the tail.

  – kTpQCD_v20 is a pQCD code is misses the low $p_T$ body part.