Estimating nuclear matter parameters from compact star observables

G.G. Barnaföldi,
D. Alvarez-Castillo, A. Ayrian, H. Grigorian, A. Jakovác, P. Pósfay, B. Szigeti

References: arXiv: 2006.03676 & 2006.03710 (EPJ ST Dec 2020), 2004.08230

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20th Zimányi Wigner School, Budapest, 11th December 2020

BREAKING NEWS

ALICE

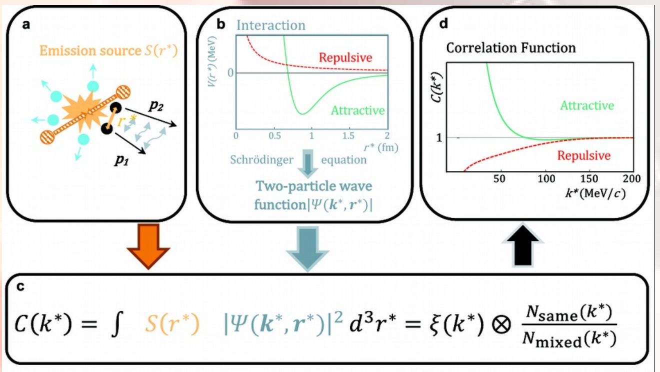
nature

Article | Published: 09 December 2020

Unveiling the strong interaction among hadrons at the LHC

ALICE Collaboration

Nature **588**, 232–238(2020) | Cite this article



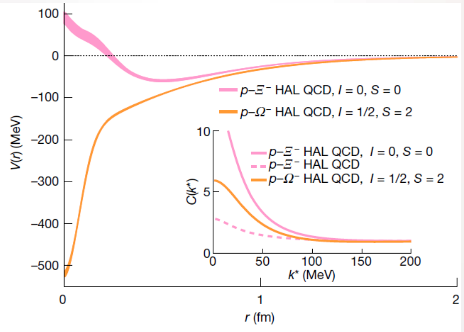
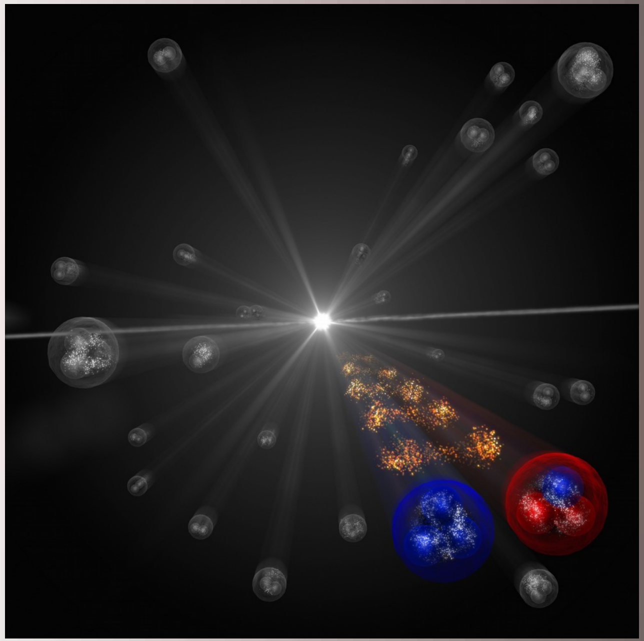
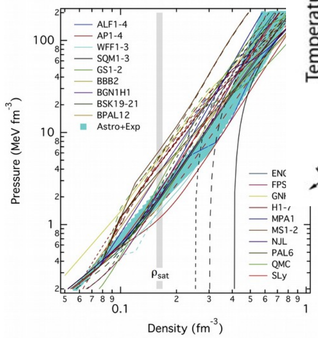


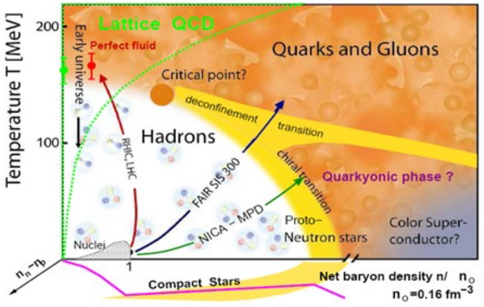
Fig. 4 | Potentials for the p–Ξ^- and p–Ω^- interactions. p–Ξ^- (pink) and p–Ω^-

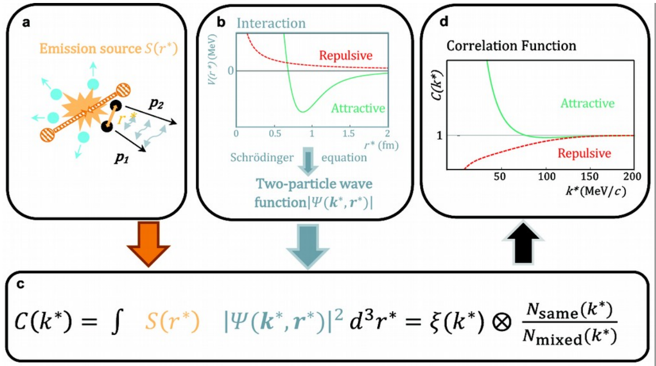


Why is this so important?

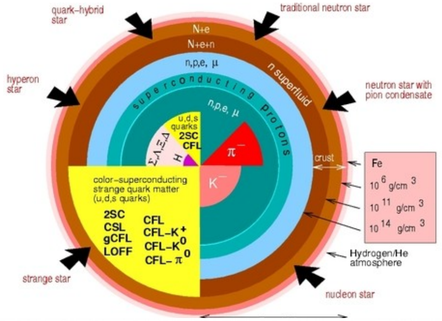
EoS experiment & theory

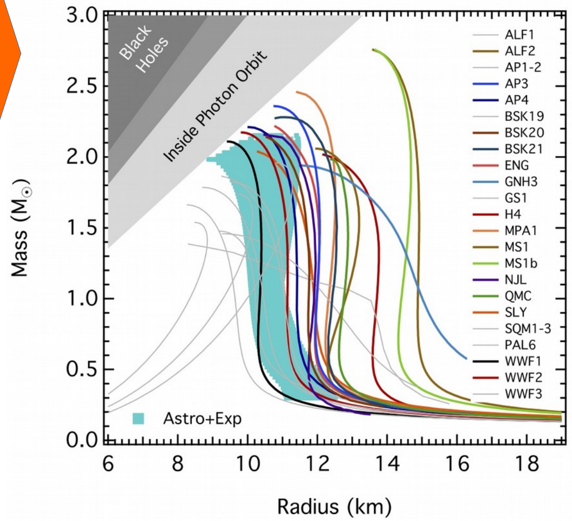




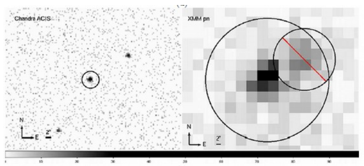


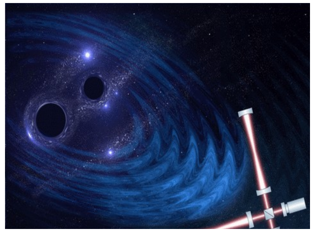
Application in compact stars

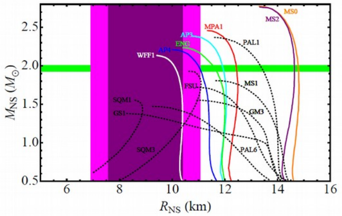


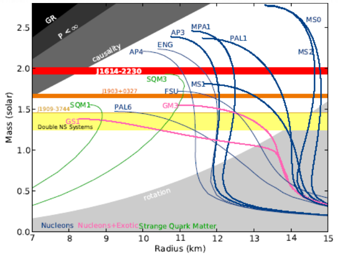


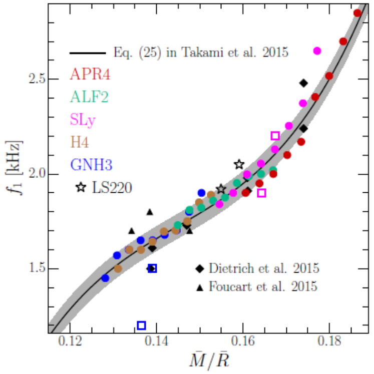
Constraints by astrophysical observations





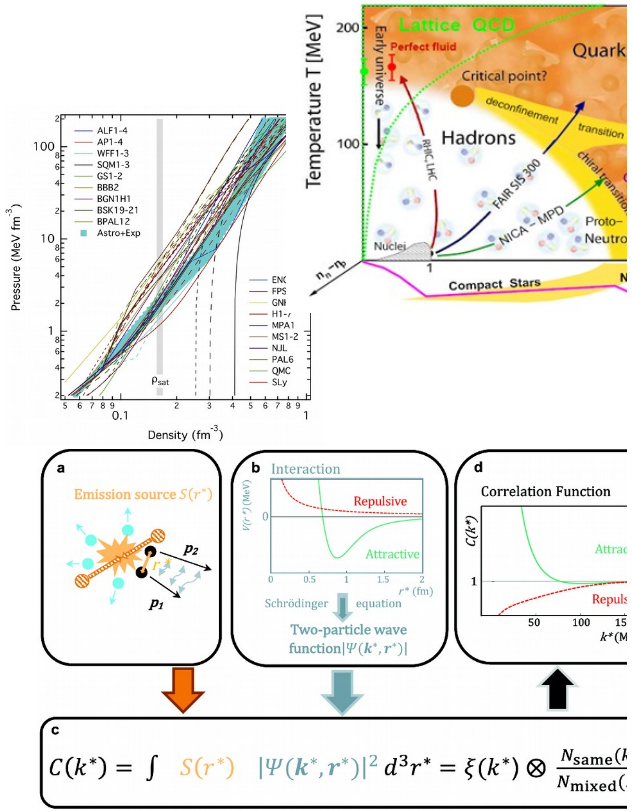




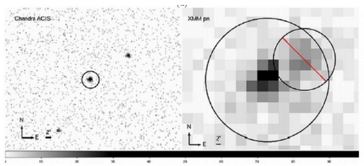


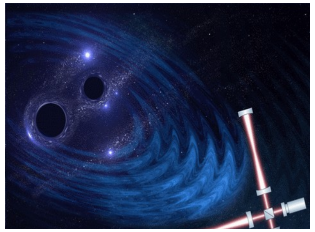
Face the masquarade problem!

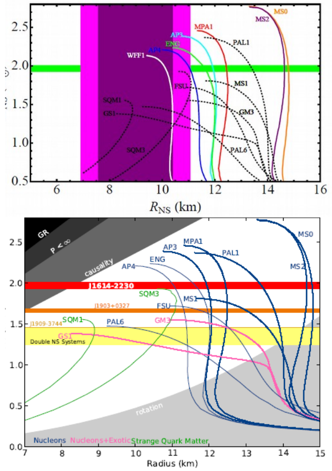
EoS
experiment & the

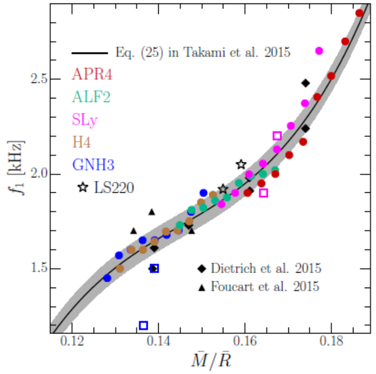


Constraints by
strophysical observations



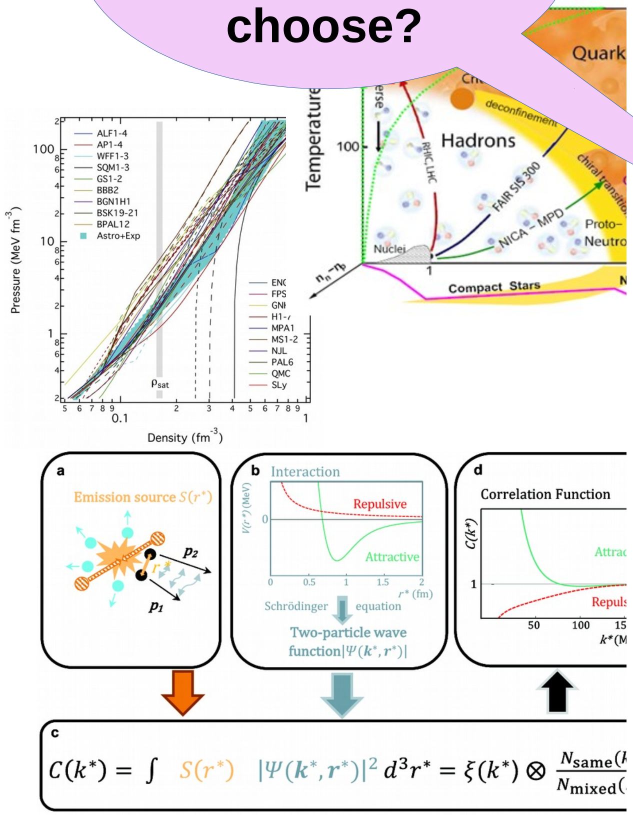






Face the masquarade problem!

Uuups, which EoS I should choose?



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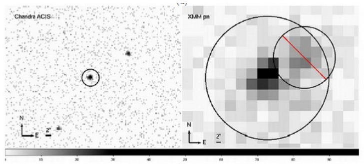
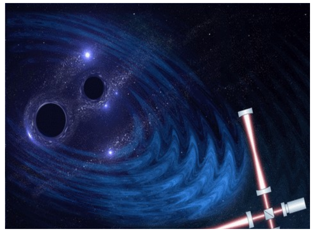
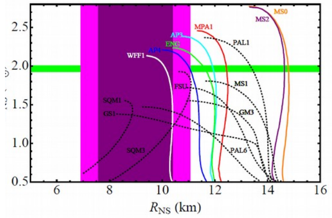
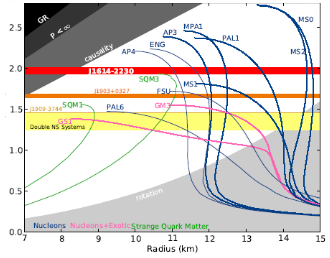
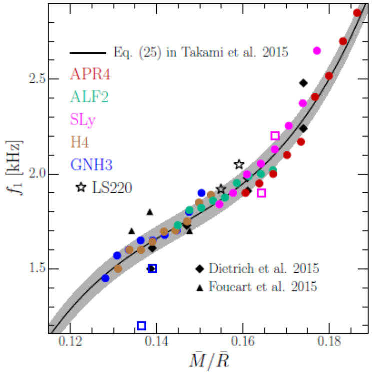
astrophysical observations











(De)motivation...

Weih & Most & Rezzolla: ApJ 881,73 (2019)

Optimal neutron-star mass ranges to constrain the equation of state of nuclear matter with electromagnetic and gravitational-wave observations

L. R. Weih,[1] E. R. Most,[1] and L. Rezzolla[1]

[1] *Institut für Theoretische Physik, Goethe Universität Frankfurt am Main, Germany*

ABSTRACT

Exploiting a very large library of physically plausible equations of state (EOSs) containing more than 10^7 members and yielding more than 10^9 stellar models, we conduct a survey of the impact that a neutron-star radius measurement via electromagnetic observations can have on the EOS of nuclear matter. Such measurements are soon to be expected from the ongoing NICER mission and will complement the constraints on the EOS from gravitational-wave detections. Thanks to the large statistical range of our EOS library, we can obtain a first quantitative estimate of the commonly made assumption that the high-density part of the EOS is best constrained when measuring the radius of the most massive, albeit rare, neutron stars with masses $M \gtrsim 2.1\, M_\odot$. At the same time, we find that radius measurements of neutron stars with masses $M \simeq 1.7 - 1.85\, M_\odot$ can provide the strongest constraints on the low-density part of the EOS. Finally, we quantify how radius measurements by future missions can further improve our understanding of the EOS of matter at nuclear densities.

The sad reality is...

Weih & Most & Rezzolla: ApJ 881,73 (2

Optimal neutron-star mass ranges to ... tter with electromagnetic and

...many

Exploit ... containing more
than 10^7 ... of the impact
that a neu ... n the EOS of
nuclear mat ... mission and
will complem ... to the large
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assumption tha ... ng the radius of
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constraints on the ... quantify how radius measurements by
future missions can ... of the EOS of matter at nuclear densities.

Let’s explore the uncertainties…

…in a traditional way

P. Pósfay, GGB, A. Jakovác: 2004.08230 (submitted to PASA), +B. Szigeti 2006.03710 (in press in EPJ ST)

Investigate this with extended σ-ω model in mean field

$$\mathcal{L}_{MF} = \sum_{i=1,2} \bar{\psi}_i \left(i\partial\!\!\!/ - m_N + g_\sigma \overline{\sigma} - g_\omega \gamma^0 \overline{\omega}_0 \right) \psi_i$$

Nucleon effective mass

Proton and neutron

$$-\frac{1}{2} m_\sigma^2 \overline{\sigma}^2 - \lambda_3 \overline{\sigma}^3 - \lambda_4 \overline{\sigma}^4$$

Scalar meson self interaction terms

$$+\frac{1}{2} m_\omega^2 \overline{\omega}_0^2$$

Extra terms

Vector meson

$$+\frac{1}{2} m_\rho^2 \rho_\mu^a \rho^{\mu\, a}$$

Tensor meson

$$+\overline{\Psi}_e \left(i\partial\!\!\!/ - m_e \right) \Psi_e$$

Electron in β-equilibrium

$$\mu_n = \mu_p + \mu_e$$

Investigate this with extended σ-ω model in mean field

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Scalar meson self interaction terms

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Vector meson

Extra terms

p – n Nuclear force

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Tensor meson

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Isospin asymmetry

Tensor meson

$$+\overline{\Psi}_e \left(i\partial\!\!\!/ - m_e \right) \Psi_e$$

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Electron in β-equilibrium

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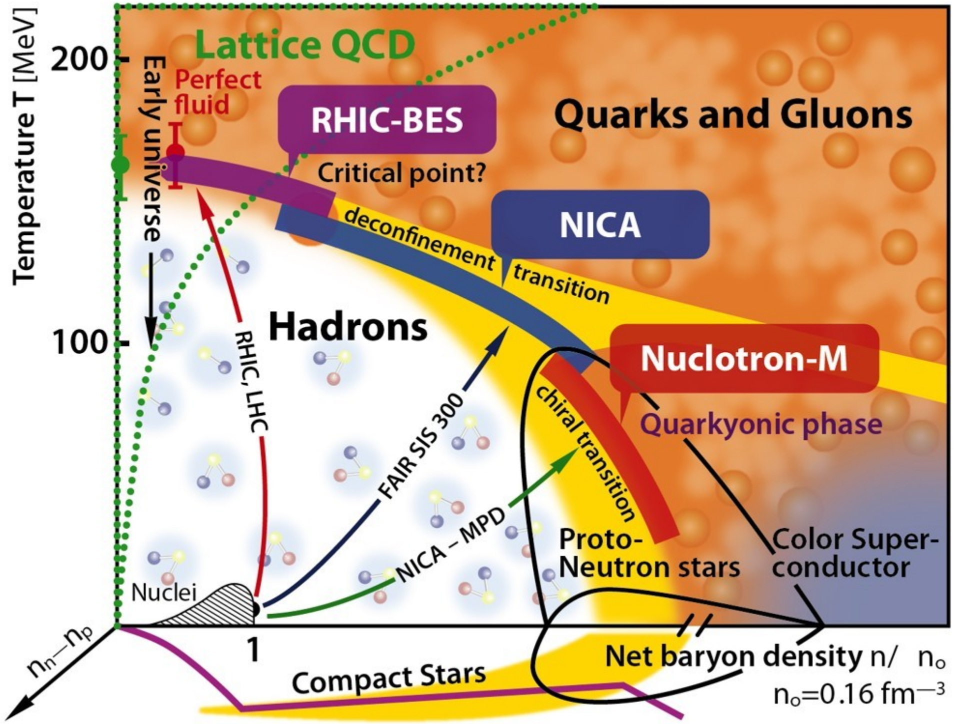
Modified σ-ω model in mean field

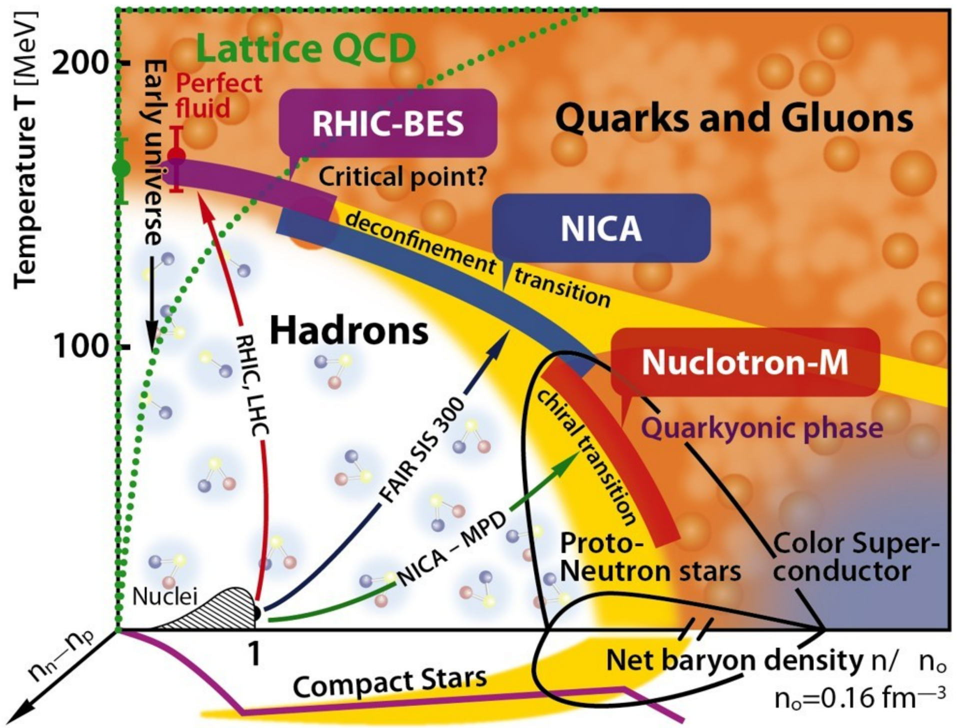
- **Theoretical mean field model:**
  - Symmetric case: 3 combinations with the higher-order scalar meson self-interaction terms to original Walecka:
  - Asymmetric case: tensor force is added to the interaction in addition to the electrons, for β-equilibrium.
- **Parameters of the theoretical model**
  - Fit couplings/masses/etc. according to the Rhoades–Ruffini theorem in agreement with experimental data.
  - Parameters are usually non-independent: optimalization of the parameters need to perform → similar EoS
- **Cross check the consistency with the the existing EM, GR, HIC, etc data + errors → Theoretical uncertainties**

Parameters to fit normal nuclear matter



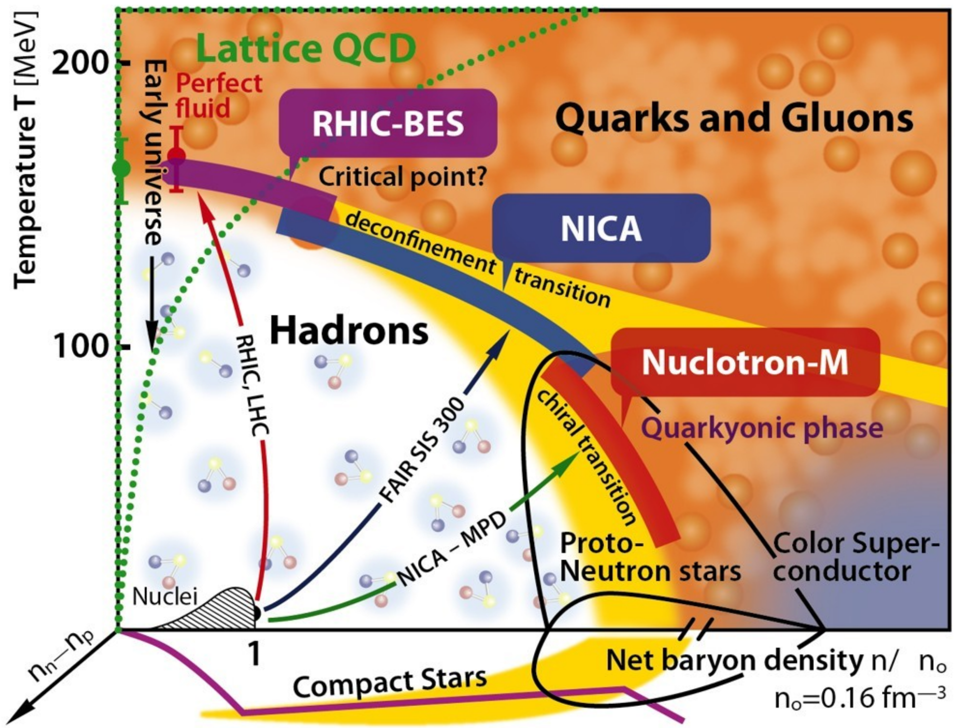
Model	ns [fm^{-3}]	B [MeV]	K [MeV]	S_0 [MeV]	m^* [m_N]
NLρ	0.1459	−16.062	203.3	30.8	0.603
NL$\rho\delta$	0.1459	−16.062	203.3	31.0	0.603
DBHF	0.1810	−16.150	230.0	34.4	0.678
DD	0.1487	−16.021	240.0	32.0	0.565
D3C	0.1510	−15.981	232.5	31.9	0.541
KVR	0.1600	−15.800	250.0	28.8	0.805
KVOR	0.1600	−16.000	275.0	32.9	0.800
DD-F	0.1469	−16.024	223.1	31.6	0.556

Parameters to fit normal nuclear matter



Parameter	Value
Saturation density	0.156 1/fm^3
Binding energy	-16.3 MeV
Nucleon effective mass	0.6 m_N
Nucleon Landau mass	0.83 m_N
incompressibility	240 MeV
Asymmetry energy	32.5 MeV

Parameters to fit normal nuclear matter



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Incompressibility $$K = k_F^2 \frac{\partial^2(\epsilon/n)}{\partial k_F^2} = 9\,\frac{\partial p}{\partial n}$$

Landau mass $$m_L = \frac{k_F}{v_F} = \sqrt{k_F^2 + m_{N,eff}^2}$$

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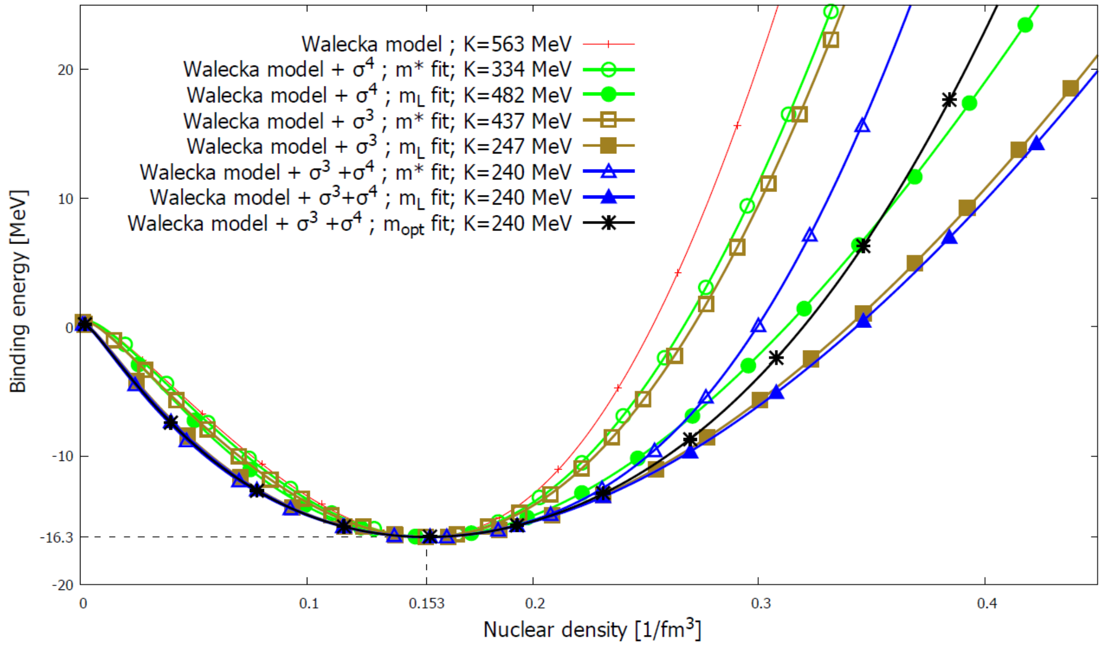

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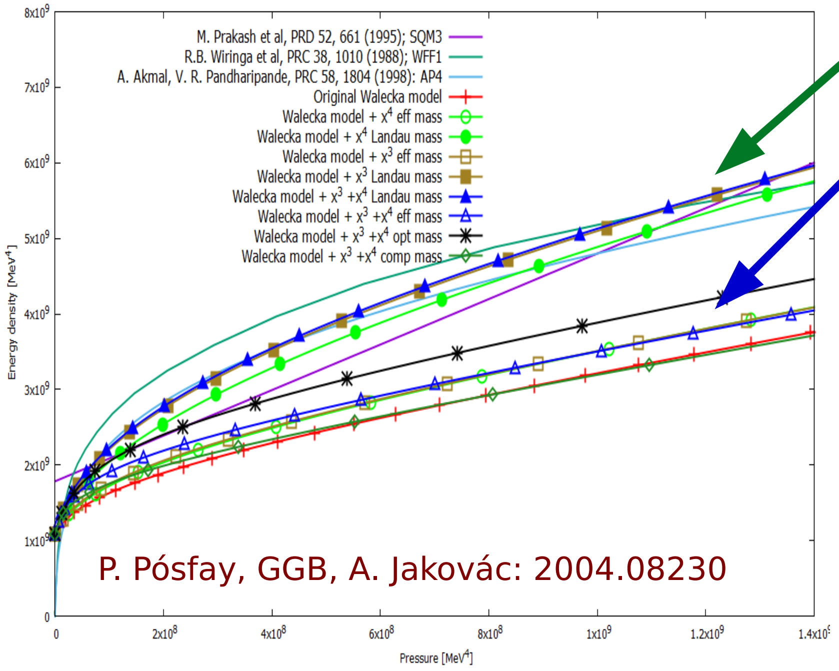
Incompressibility $$K = k_F^2 \frac{\partial^2(\epsilon/n)}{\partial k_F^2} = 9\,\frac{\partial p}{\partial n}$$

The effective mass and Landau mass are NOT independent!
The can not be fitted simultaneously

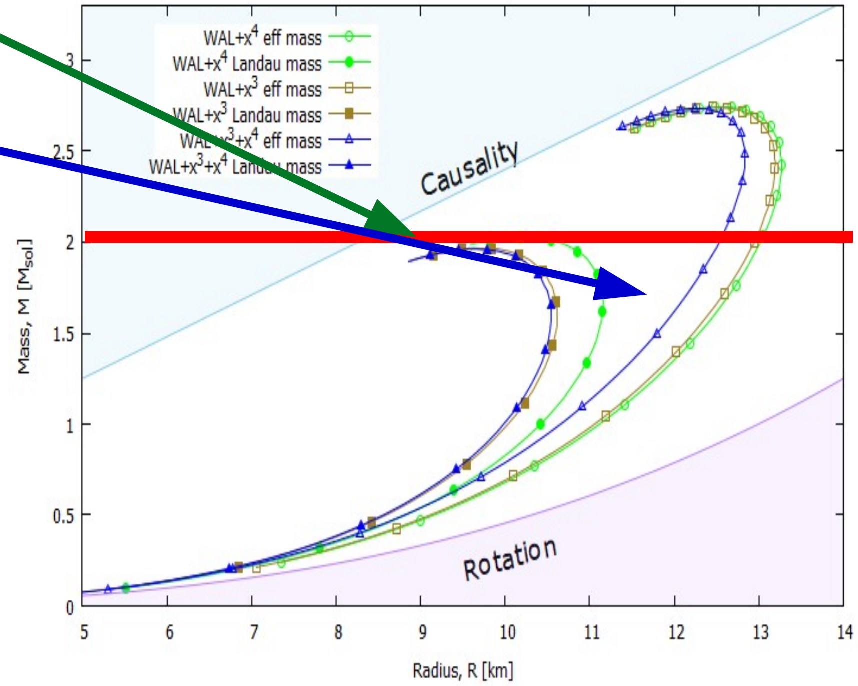
Landau mass $$m_L = \frac{k_F}{v_F} = \sqrt{k_F^2 + m_{N,eff}^2}$$

The EoS & M-R of different model fits



Landau mass fit $m_{Eff} = 0.83\ m_N$

Effective mass fit $m_{Eff} = 0.6\ m_N$

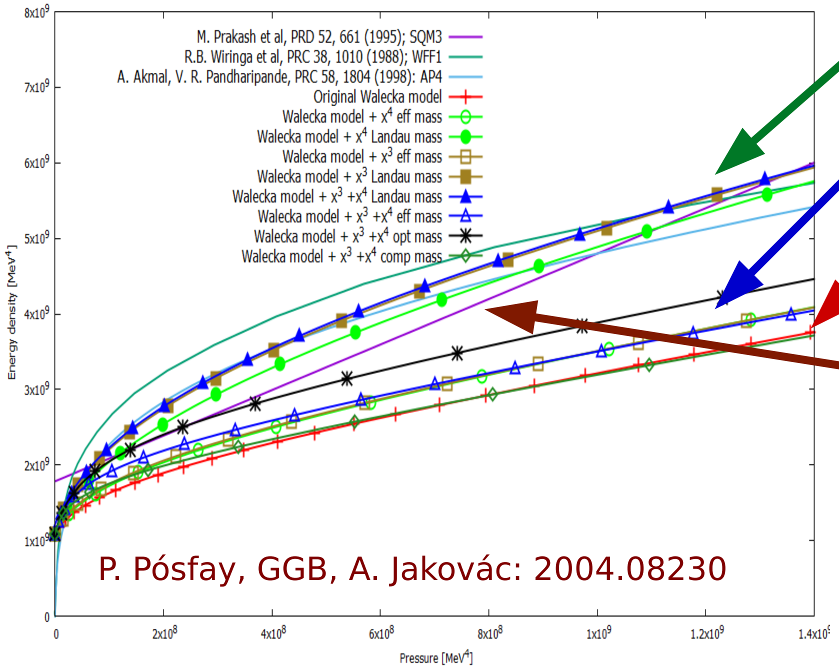


M-R diagram with these nuclear matter EoS

Cases with extra x^3 and/or x^4 terms provide similar band structures

→ Landau mass fits provide lower M_{max} closer to the observations

The EoS & M-R of different model fits



P. Pósfay, GGB, A. Jakovác: 2004.08230

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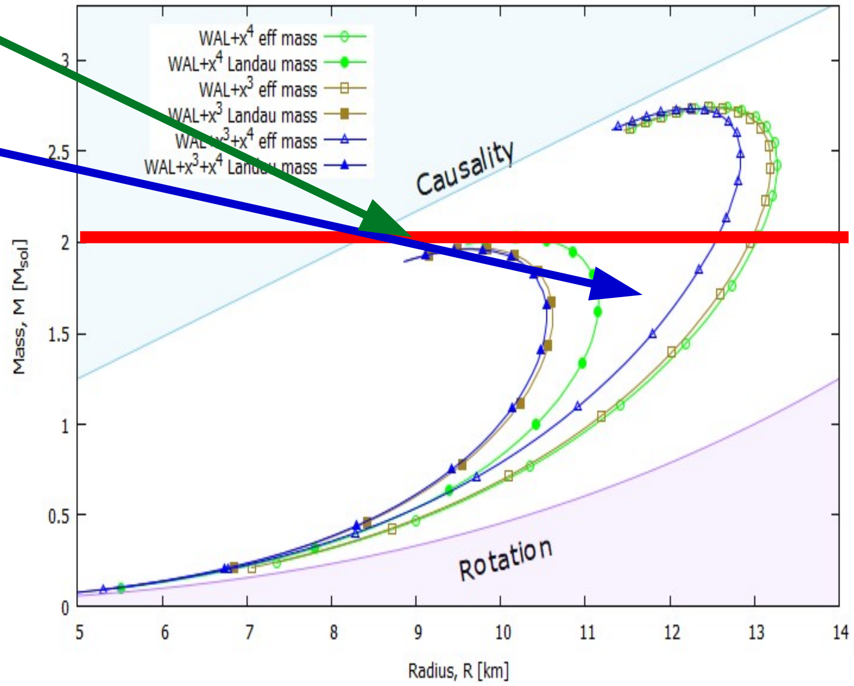
Original Walecka model

Realistic nuclear EoS: WFF1, AP4 (SQM) support the Landau mass



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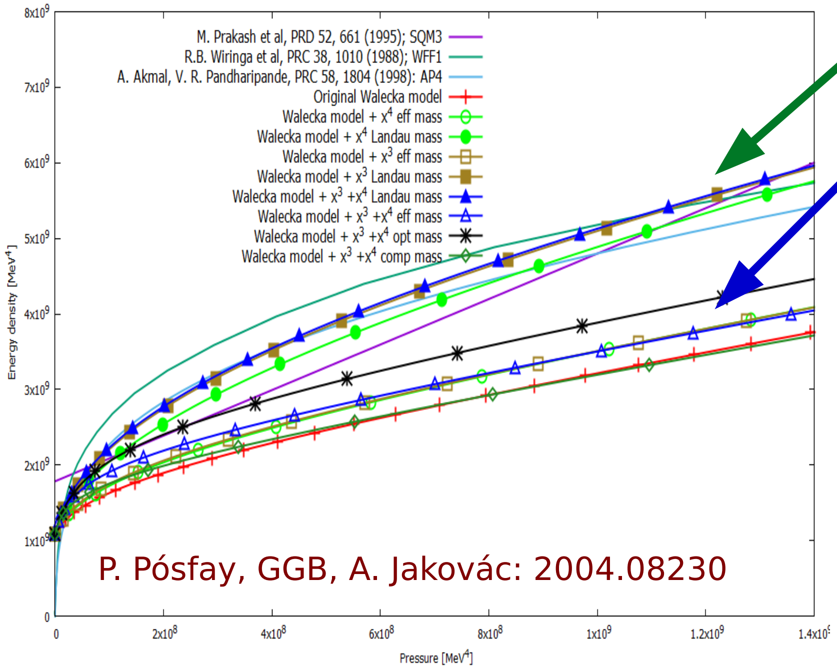

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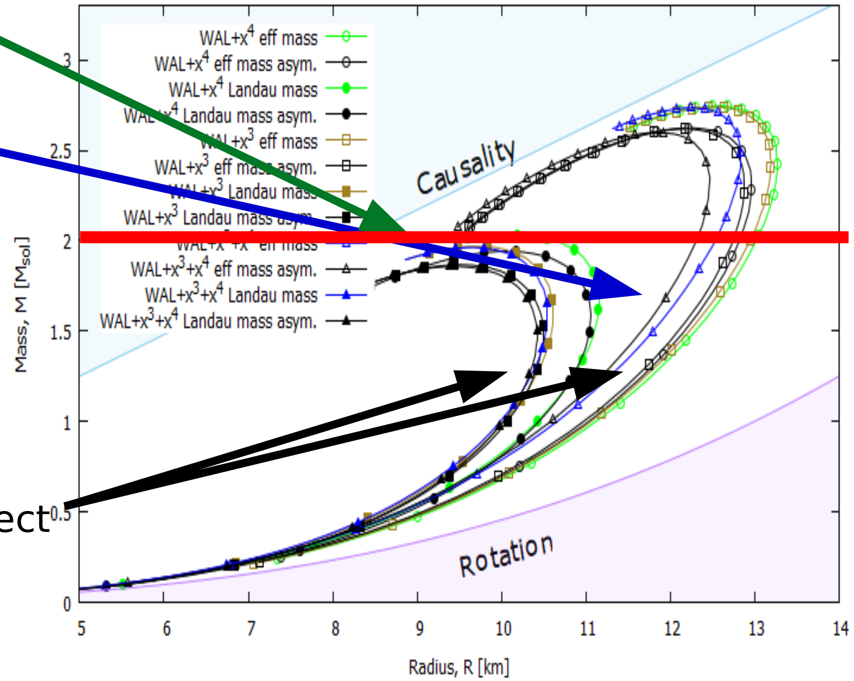
Assymetry (electrons) is weak effect



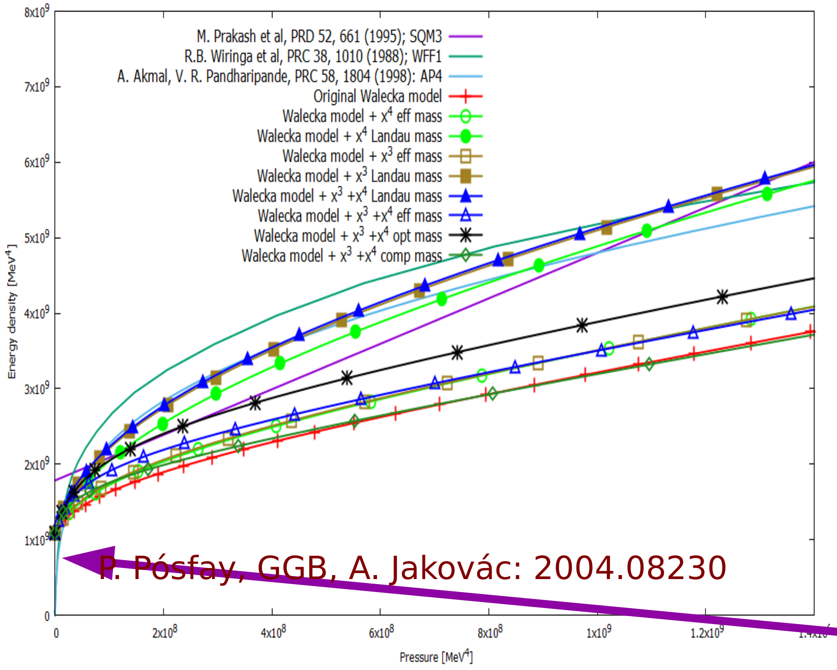
M-R diagram with these nuclear matter EoS

Cases with extra x^3 and/or x^4 terms provide similar band structures

→ Landau mass fits provide lower M_{max} closer to the observations

→ Nuclear ASYMMETRY result in 10-20% lower M_{max}

The EoS & M-R of different model fits



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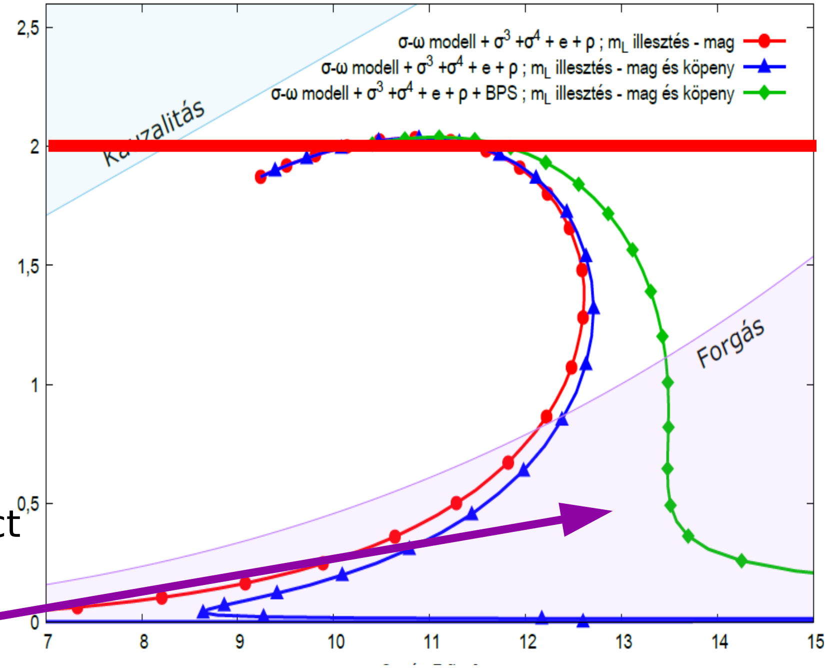
Original Walecka model

Realistic nuclear EoS: WFF1, AP4 (SQM) support the Landau mass

Assymetry (electrons) is weak effect

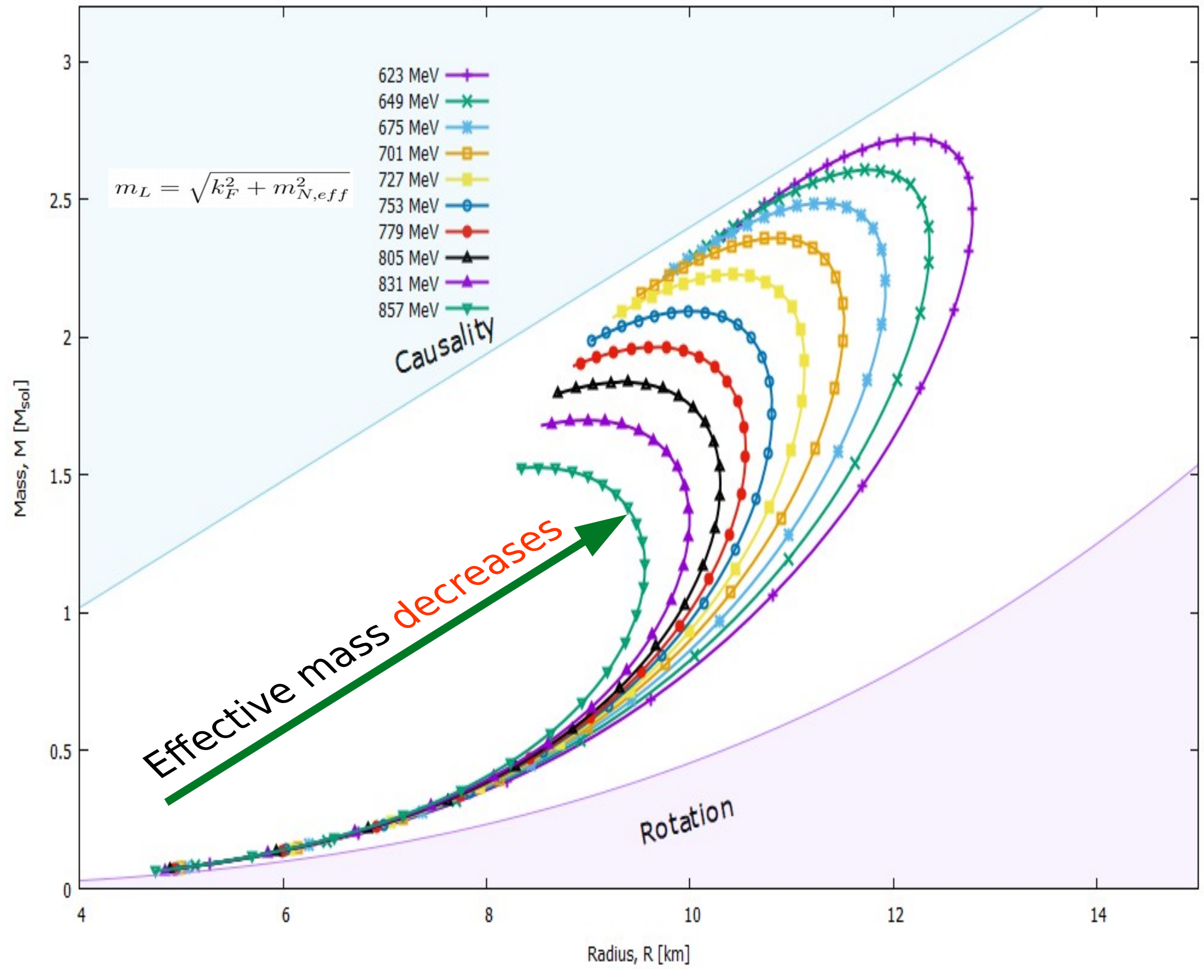
Crust (BPS) make more realistic



M-R diagram with these nuclear matter EoS

- Cases with extra x^3 and/or x^4 terms provide similar band structures
- → Landau mass fits provide lower M_{max} closer to the observations
- → Nuclear ASYMMETRY result in 10-20% lower M_{max}
- → Adding CORE with BPS has no effect on M_{max}, only on R (~km)

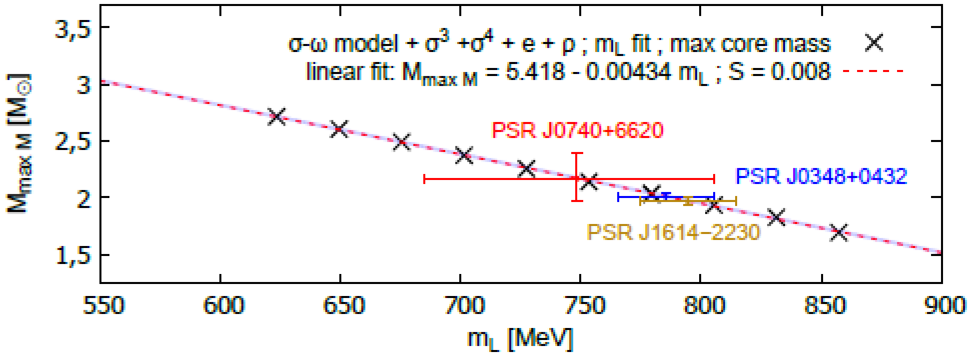
The M-R diagrams: EoS & Landau mass fit



Evolution/scaling in M_{max} appears

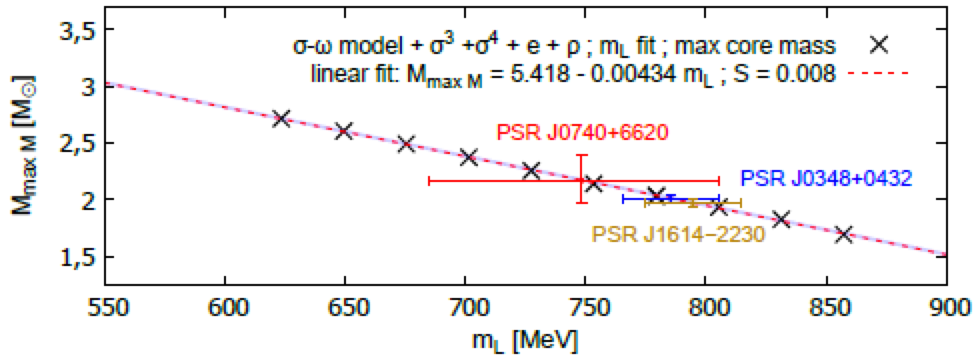
- The M_{max} is increasing as the Landau (effective) mass is decreasing

→ Scaling by nuclear parameters



P. Pósfay, GGB, A. Jakovác: 2004.08230

Scaling: maximum star mass vs. nuclear parameters

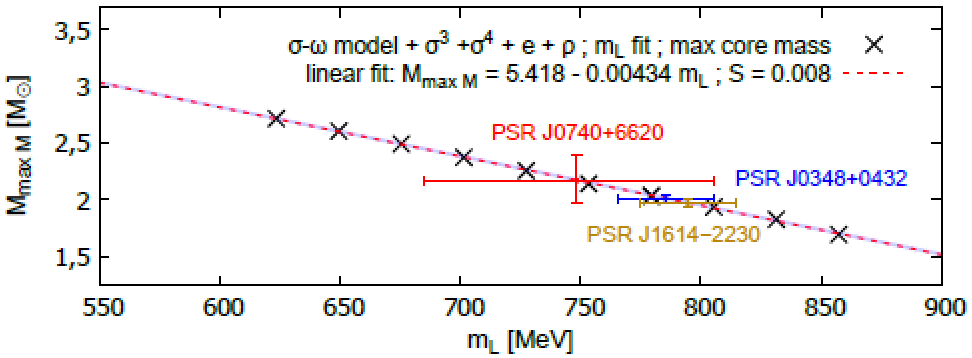


Maximal mass
with Landau mass

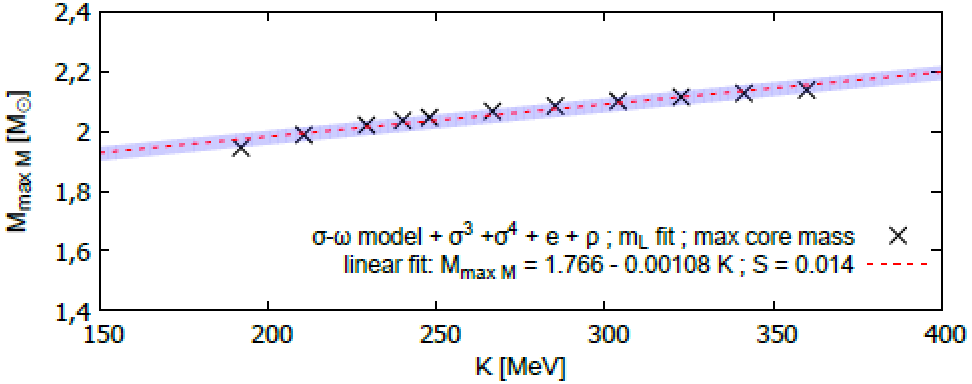
$$M_{maxM}(m_L)[\mathrm{M}_\odot] = 5.418 - 0.00434\, m_L[\mathrm{MeV}]$$

P. Pósfay, GGB, A. Jakovác: 2004.08230

Scaling: maximum star mass vs. nuclear parameters



a)



Maximal mass
with Landau mass

$$M_{maxM}(m_L)[\mathrm{M}_\odot] = 5.418 - 0.00434\, m_L\,[\mathrm{MeV}]$$

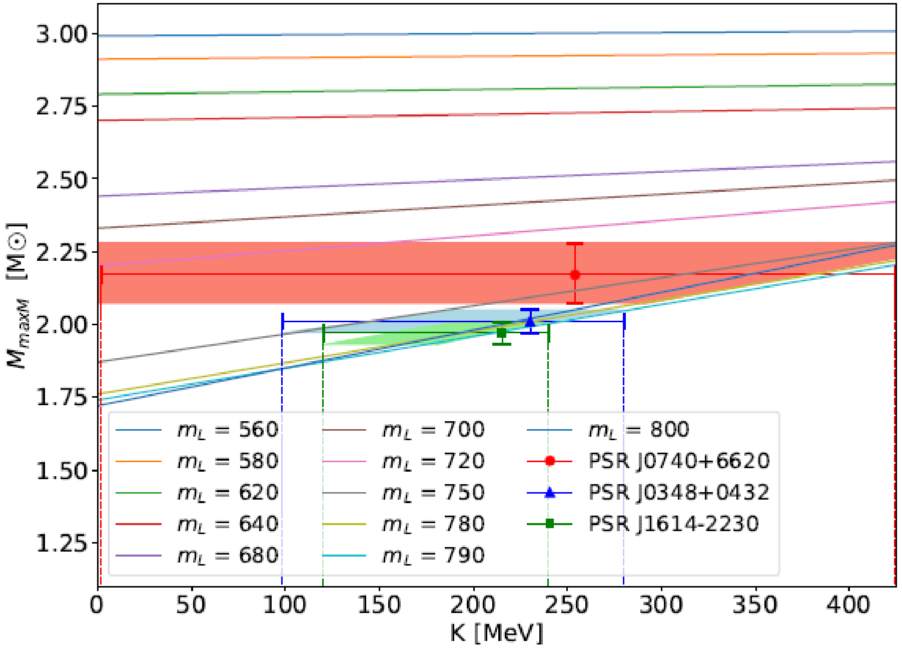
with (in)compressibility

$$M_{maxM}(K)[\mathrm{M}_\odot] = 1.766 + 0.00110K\ [\mathrm{MeV}]$$

$$\Delta M_{max}(\delta m_L) \overset{10\times}{>} \Delta M_{max}(\delta K)$$

P. Pósfay, GGB, A. Jakovác: 2004.08230

Scaling: maximum star mass vs. nuclear parameters



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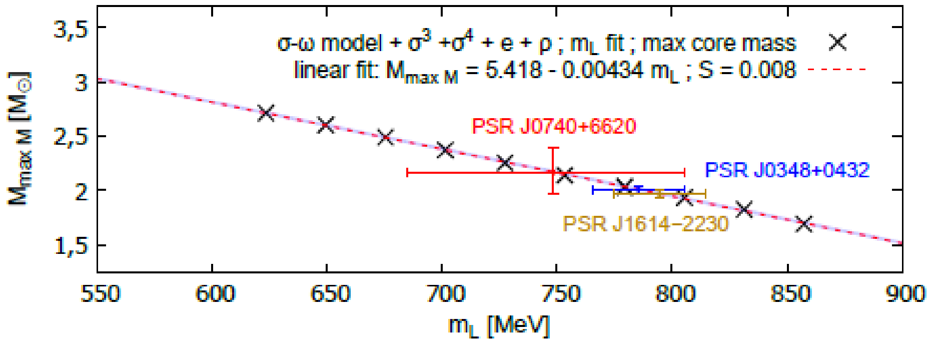
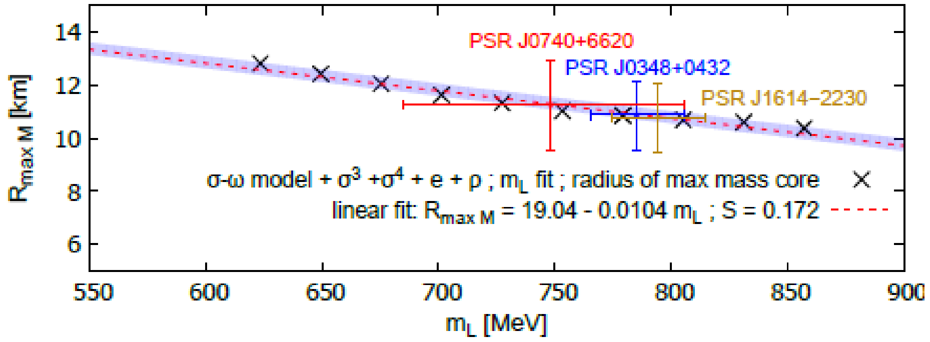
$$M_{maxM}(m_L, K)[\mathrm{M}_\odot] = 6.29 - 0.00574\, m_L\ [\mathrm{MeV}] - 0.00379\, K\ [\mathrm{MeV}] + 0.00000524\, m_L \cdot K\ [\mathrm{MeV}^2]\, ,$$

P. Pósfay, GGB, A. Jakovác, B. Szigeti 2006.03710

Scaling: maximum star mass vs. nuclear parameters





Maximal mass & its radius with Landau mass

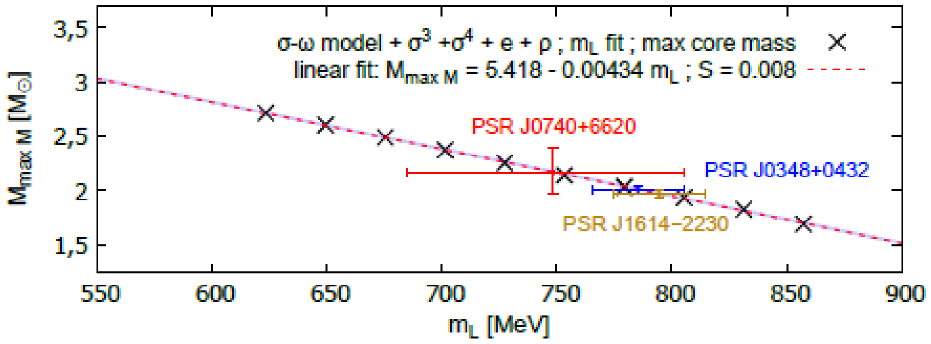
$$M_{maxM}(m_L)[\mathrm{M}_\odot] = 5.418 - 0.00434\, m_L\,[\mathrm{MeV}]$$
$$R_{maxM}(m_L)[\mathrm{km}] = 19.04 - 0.01040\, m_L\,[\mathrm{MeV}]$$

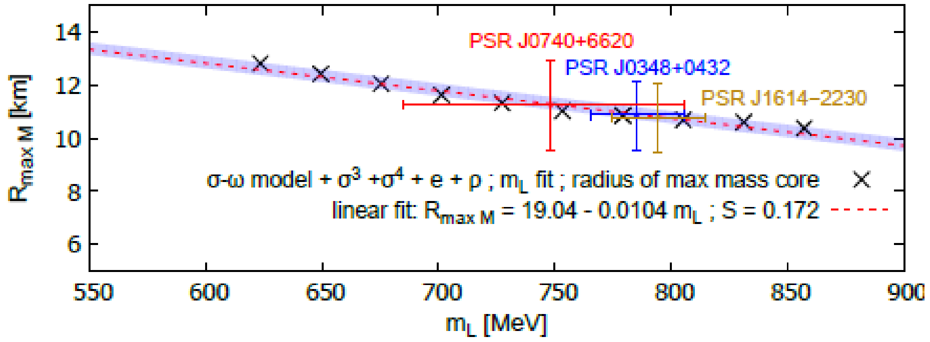
with (in)compressibility

$$M_{maxM}(K)[\mathrm{M}_\odot] = 1.766 + 0.00110K\ [\mathrm{MeV}]$$
$$R_{maxM}(K)[\mathrm{km}] = 8.878 + 0.00767K\ [\mathrm{MeV}]$$

P. Pósfay, GGB, A. Jakovác, B. Szigeti 2006.03710

Scaling: maximum star mass vs. nuclear parameters





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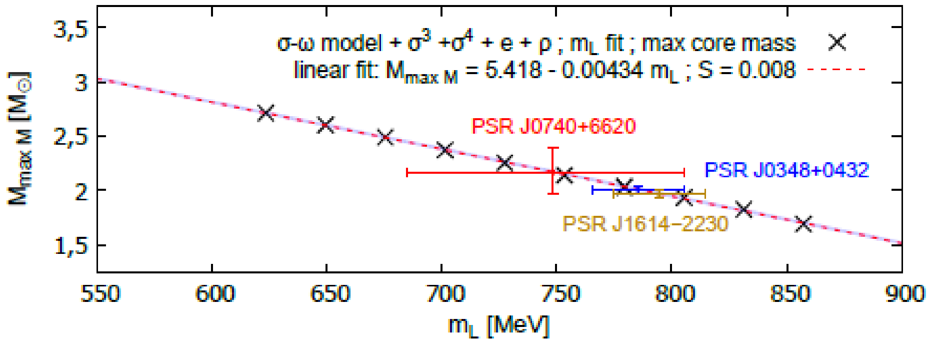
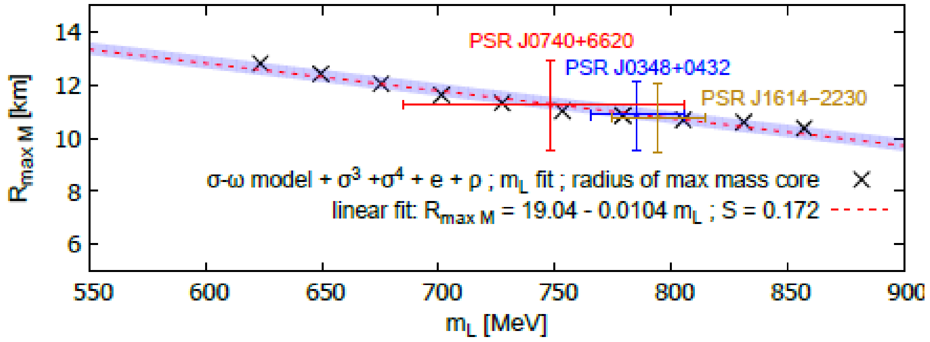
Calculation for maximal mass star

Measured: $M_{maxM} \rightarrow (m_L\ \&\ K) \rightarrow R_{maxM}$

P. Pósfay, GGB, A. Jakovác, B. Szigeti 2006.03710

Scaling: maximum star mass vs. nuclear parameters





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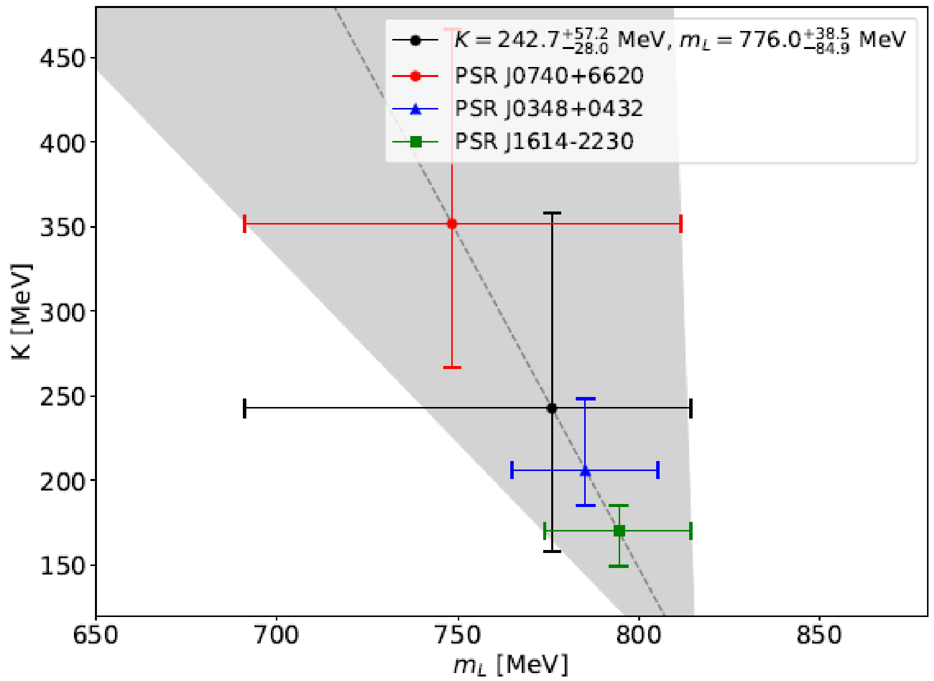
Calculation for maximal mass star

Pulsar	R_{maxM} [km]	M_{maxM} [$\mathrm{M}_\odot$]	m_L [MeV]	K [MeV]
PSR J0740+6620	$11.25^{+1.06}_{-1.04}$	$2.17^{+0.11}_{-0.10}$ *	$748.39^{+63.3}_{-57.2}$	$351.8^{+115}_{-84.5}$
PSR J0348+0432	$10.87^{+0.82}_{-0.80}$	$2.01^{+0.04}_{-0.04}$ *	$785.25^{+20.0}_{-20.3}$	$206.4^{+42.7}_{-20.5}$
PSR J1614−2230	$10.77^{+0.82}_{-0.80}$	$1.97^{+0.04}_{-0.04}$ *	$794.47^{+20.1}_{-20.4}$	$170.0^{+15.5}_{-20.9}$

P. Pósfay, GGB, A. Jakovác, B. Szigeti 2006.03710

From data: Maximum star mass vs. nuclear parameters



Maximal mass & its radius
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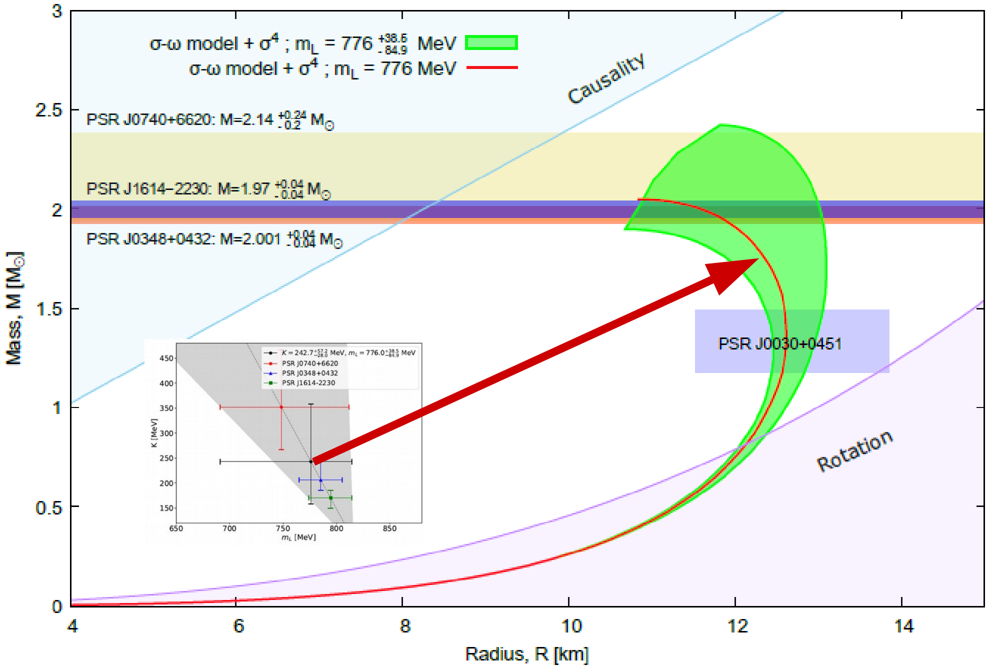
$$M_{maxM}(m_L, K)[\mathrm{M}_\odot] = 6.29 - 0.00574\, m_L\,[\mathrm{MeV}] - 0.00379\, K\,[\mathrm{MeV}] + 0.00000524\, m_L \cdot K\,[\mathrm{MeV}^2],$$

$$R_{maxM}(m_L, K)[\mathrm{km}] = 27.51 - 0.0239\, m_L\,[\mathrm{MeV}] - 0.0241\, K\,[\mathrm{MeV}] + 0.0000411\, m_L \cdot K\,[\mathrm{MeV}^2]$$

P. Pósfay, GGB, A. Jakovác, B. Szigeti 2006.03710

From data: Maximum star mass vs. nuclear parameters



Maximal mass & its radius with Landau mass

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$$R_{maxM}(K)[\mathrm{km}] = 8.878 + 0.00767K\ [\mathrm{MeV}]$$

Results from data using fit formulae:

$$m_L = 776.0^{+38.5}_{-84.9}\ \mathrm{MeV\ and}\ K = 242.7^{+57.2}_{-28.0}\ \mathrm{MeV}$$

Explore the uncertainties…

… using a the brute force

D. Alvarez-Castillo, A. Ayrian, GGB, P. Pósfay: Phys. Part & Nucl. 51, 725 (2020), +H. Grigorian 2006.0376 (in press EPJ ST)

Brute force: Bayesian analysis

Data: $\vec{\pi}_q = \{m_{L(i)}, K_{0(j)}, S_{0(k)}\}$

Likelihood for given independent constraints:

$$P(E|\vec{\pi}_q) = \prod_w P(E_w|\vec{\pi}_q)$$

Posterior:

$$P(\vec{\pi}_q|E) = \frac{P(E|\vec{\pi}_q)\,P(\vec{\pi}_q)}{\sum_{p=0}^{N-1} P(E|\vec{\pi}_p)\,P(\vec{\pi}_p)}$$

Marginalization (1-parameter):

$$P(m_{L(i)}|E) = \sum_{j,k} P(m_{L(i)}, K_{0(j)}, S_{0(k)}|E)$$

D. Alvarez-Castillo, A. Ayrian, GGB, P. Pósfay: Phys. Part & Nucl. 51, 725 (2020)

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Marginalization (1-parameter):

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Likelihood for GW170817:

$$P(E_{GW}|\pi_q) = \int_l \beta(\Lambda_1(n_c), \Lambda_2(n_c))\,\mathrm{d}n_c$$

Likelihood for maximal mass

$$P(E_M|\pi_q) = \Phi(M_q, \mu_\mathrm{C}, \sigma_\mathrm{C}) \times \Phi(M_q, \mu_\mathrm{A}, \sigma_\mathrm{A}) \times \mathcal{N}(M_q, \mu_\mathrm{U}, \sigma_\mathrm{U})$$

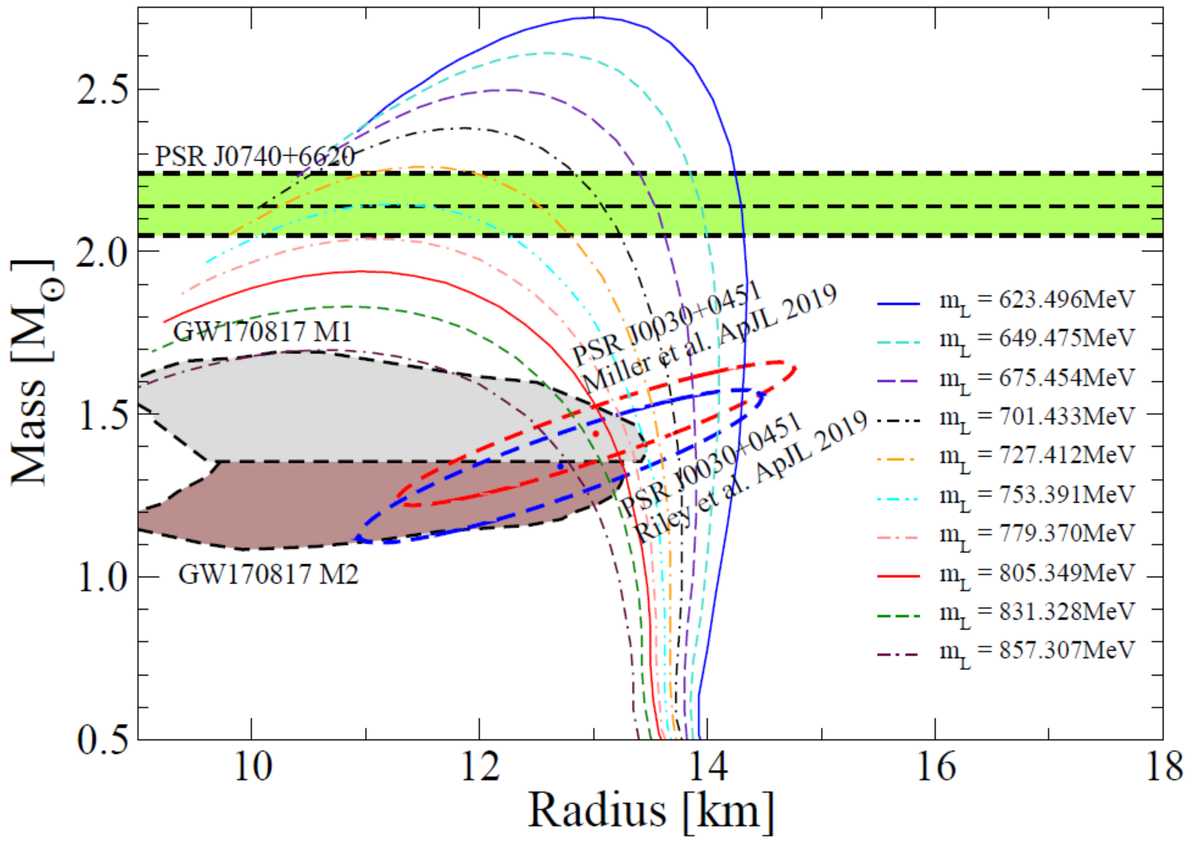
Likelihood for mass & radius

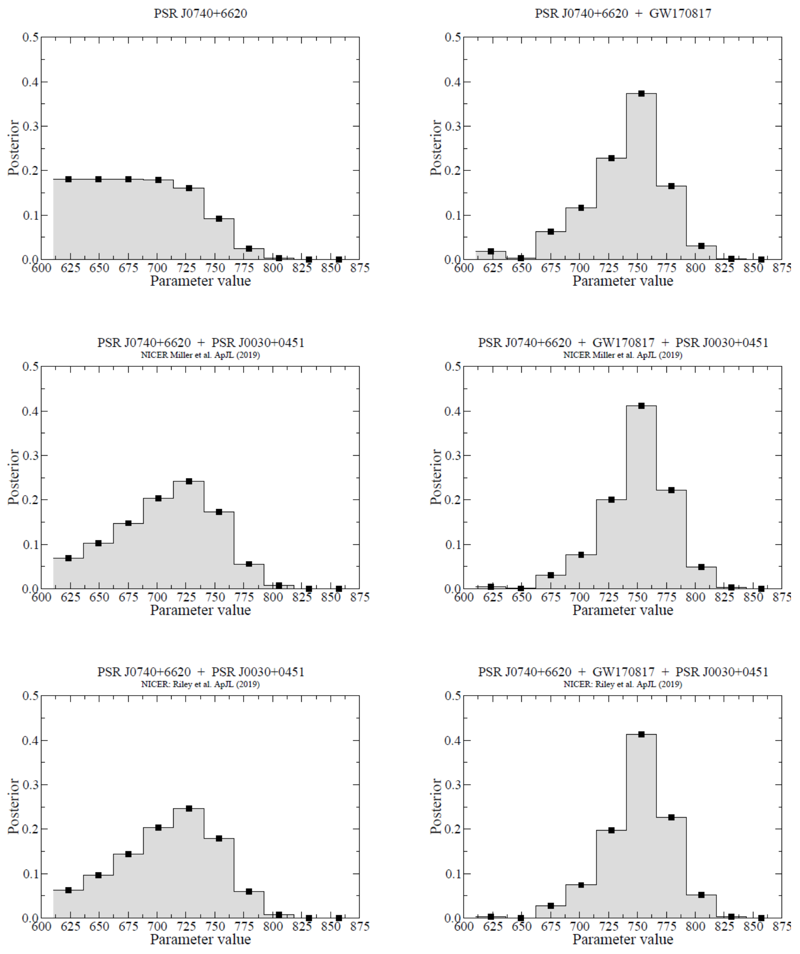
$$P(E_{MR}|\pi_q) = 0.5 \int_l \mathcal{N}\left(\mu_M^{(1)}, \sigma_M^{(1)}, \mu_R^{(1)}, \sigma_R^{(1)}, \alpha^{(1)}\right) \mathrm{d}n_c + 0.5 \int_l \mathcal{N}\left(\mu_M^{(2)}, \sigma_M^{(2)}, \mu_R^{(2)}, \sigma_R^{(2)}, \alpha^{(2)}\right) \mathrm{d}n_c,$$

D. Alvarez-Castillo, A. Ayrian, GGB, P. Pósfay: Phys. Part & Nucl. 51, 725 (2020)

Brute force: Bayesian analysis

Data with m_L only





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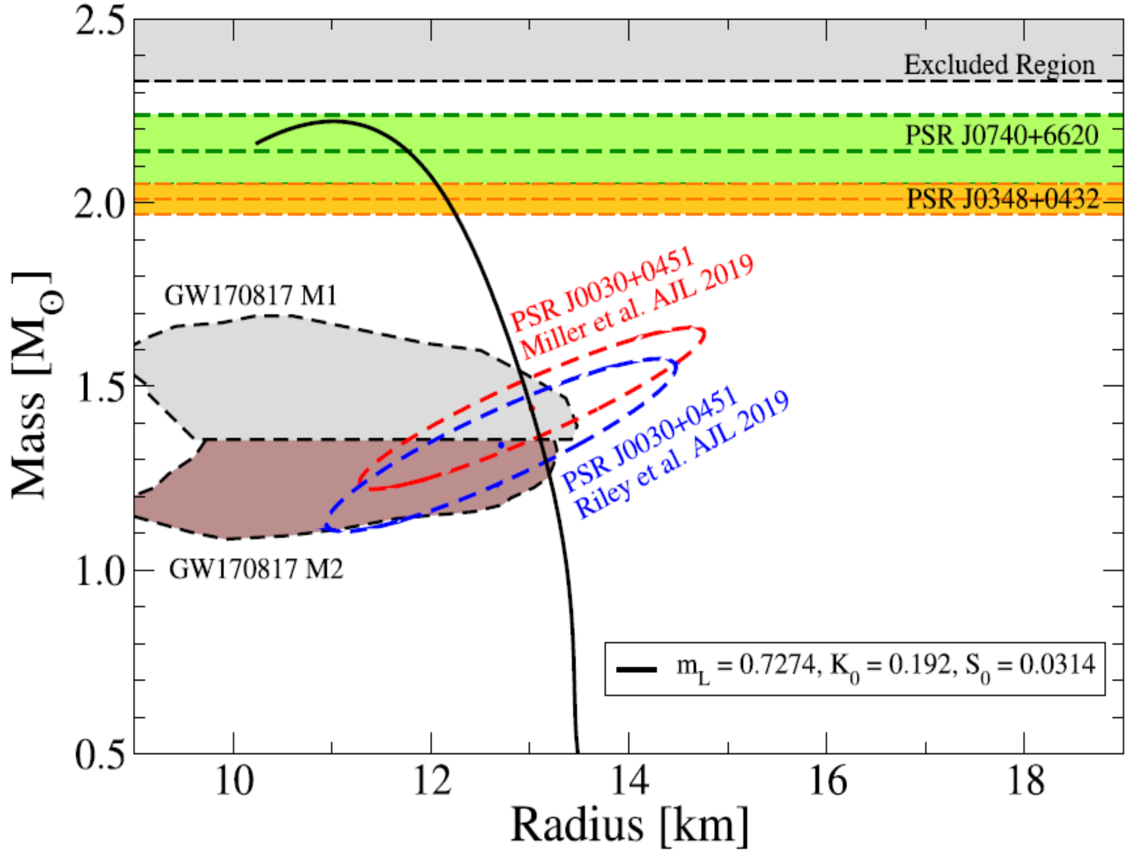
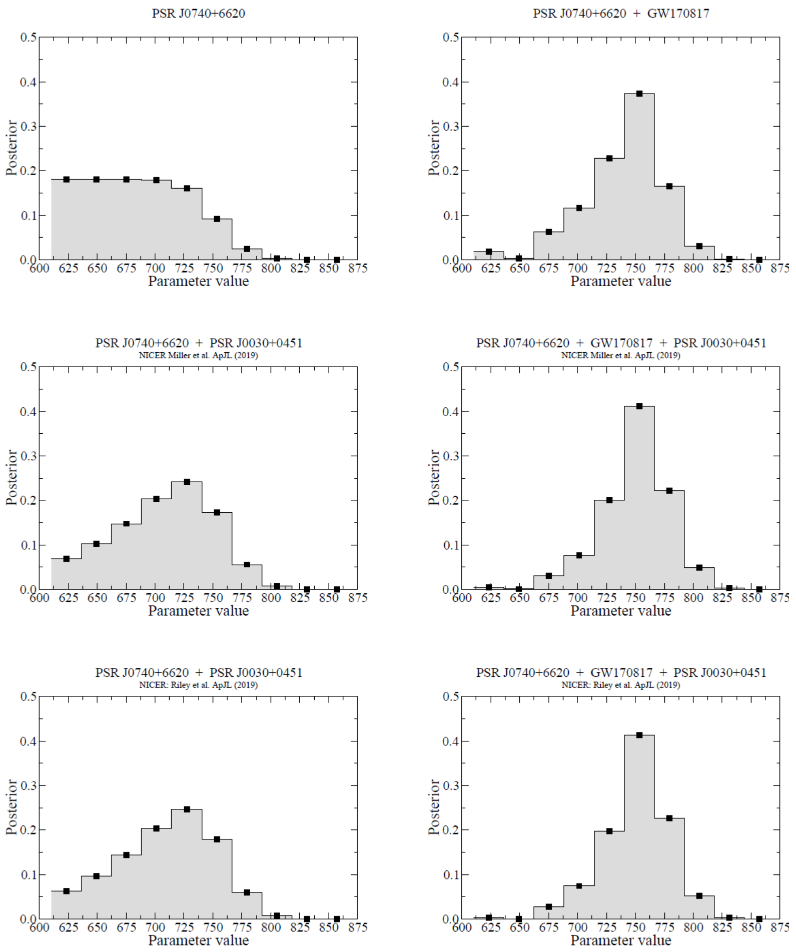
Brute force: Bayesian analysis

Data with m_L only

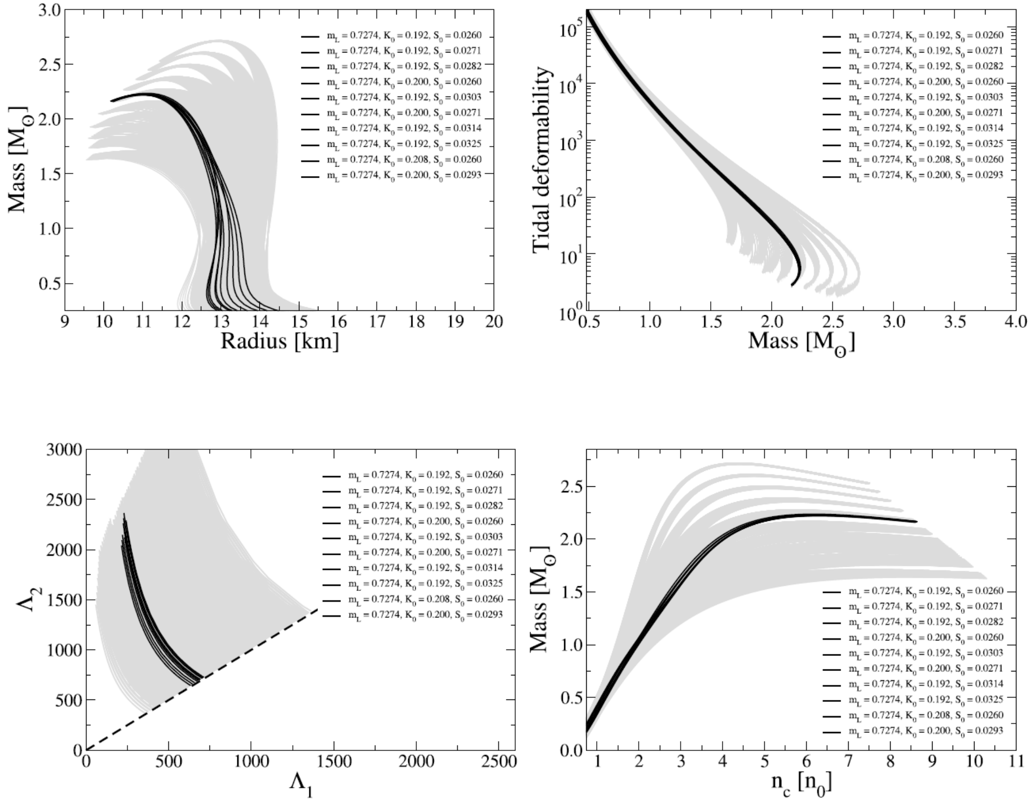


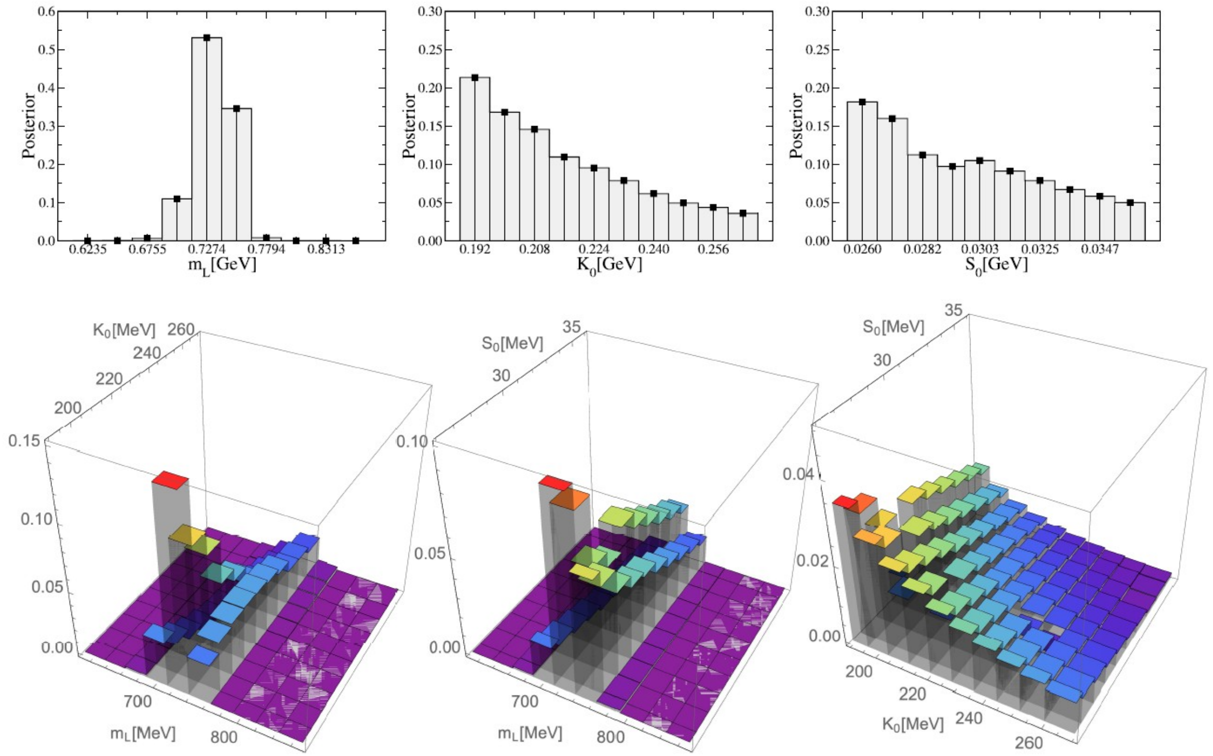


D. Alvarez-Castillo, A. Ayrian, GGB, P. Pósfay: Phys. Part & Nucl. 51, 725 (2020)

Brute force: a Bayesian analysis

Data with m_L & K





D. Alvarez-Castillo, A. Ayrian, GGB, P. Pósfay, H. Grigorian 2006.0376 (in press EPJ ST)

Summary:

- **Traditional way: mean field model**
  - In CORE approximation: maximal mass provide a unique message:
    - → strong linear Landau mass dependence
    - → an order of magnitude smaller K-dependence

$$\Delta M_{max}(\delta m_L) \overset{10\times}{>} \Delta M_{max}(\delta K) \overset{10\times}{>} \Delta M_{max}(\delta a_{sym}).$$

  - Soft part of the EoS changes the CRUST, thus vary R
  - FRG: parameter 10-25% observables: 5-10%
- **Values & uncertainties – a cross check**
  - Traditional model: $m_L = 776.0^{+38.5}_{-84.9}$ MeV and $K = 242.7^{+57.2}_{-28.0}$ MeV
  - Bayesian model*: $m_L = 727.4 \pm 15$ MeV and $K = 232 \pm 20$ MeV