Constraint of Compact Star Observables for Walecka-type Nuclear Matter EoS

Gergely Gábor Barnaföldi, Péter Pósfay, Antal Jakovác

References:
arXiv:1905.01872 [hep-th], *PASA* **35** (2018) 19, PRC **97** (2018) 025803

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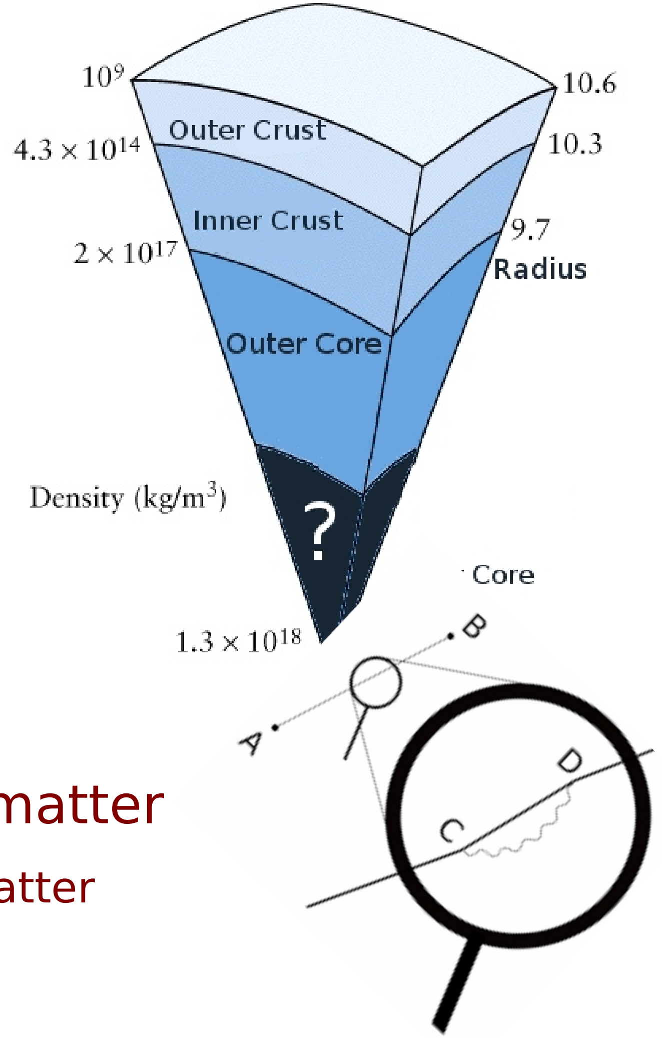
Outline

Motivation

- Predict the uncertainty of the macroscopic Compact Star observables based on the theoretical uncertainties → Masquarade problem
- How strong constraints can be obtained from Compact star measuremen



1) How much difference arise from different approximations?

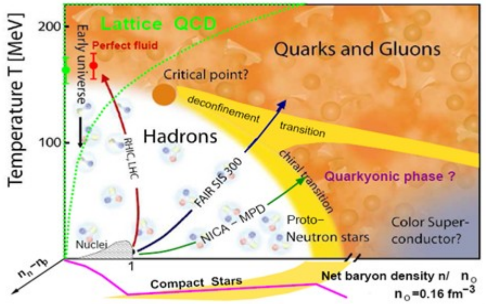
- MINIMALISTIC 1-boson-1-fermion model with a Yukawa coupling at T=0
- Uncertainties at various levels: FRG, MF and 1-loop approximations

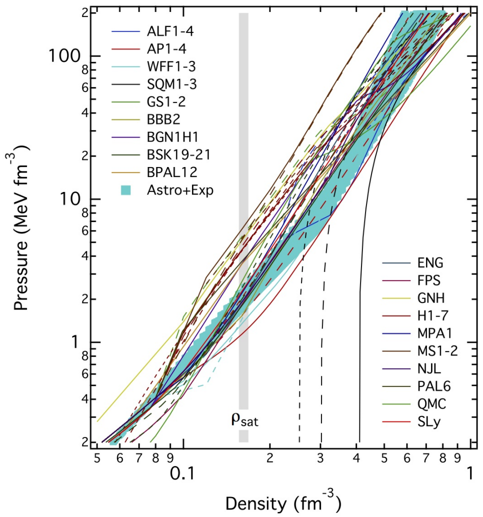
2) Uncertainties from the parameters of the realistic nuclear matter

- Parameter dependence in the extended Walecka model for symmetric matter
- Comparison between symmetric and asymmetric matter parameters

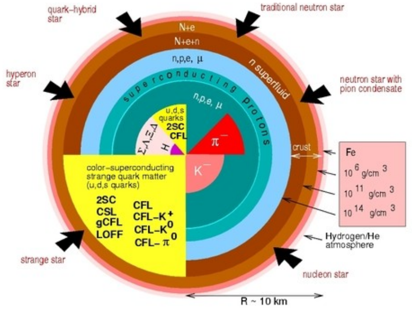
Motivation

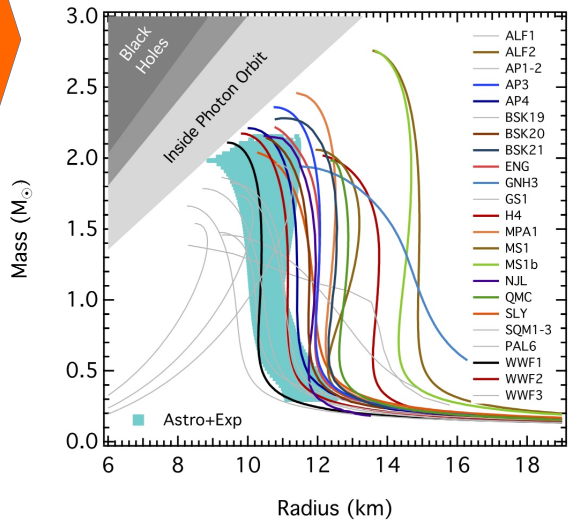
EoS from exp & theory



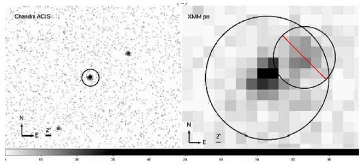


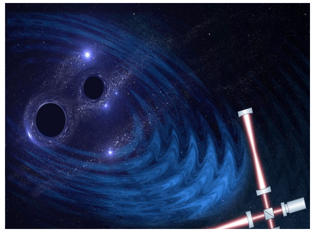
Application in compact stars

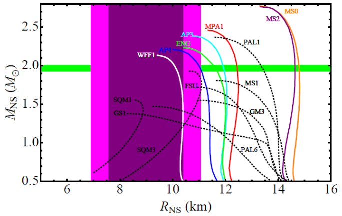


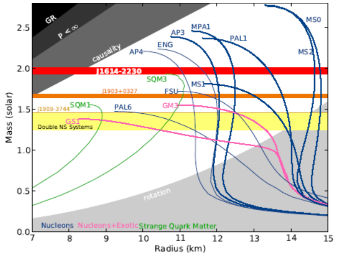


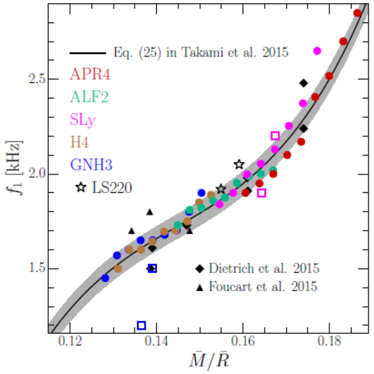
Constraints by astrophysical observations









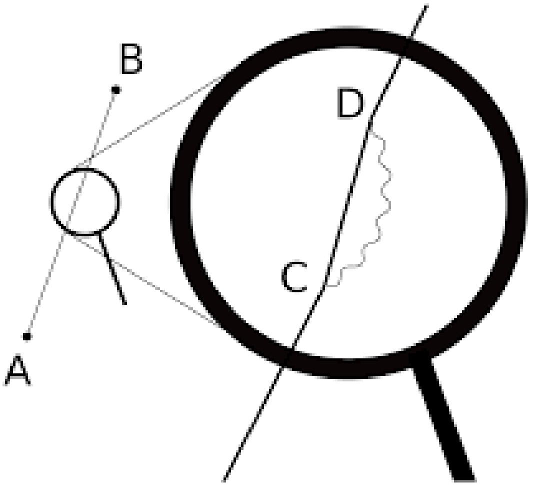


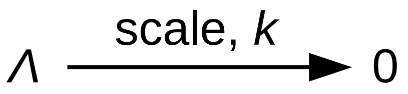
1) How much difference arise from the different levels of approximations?

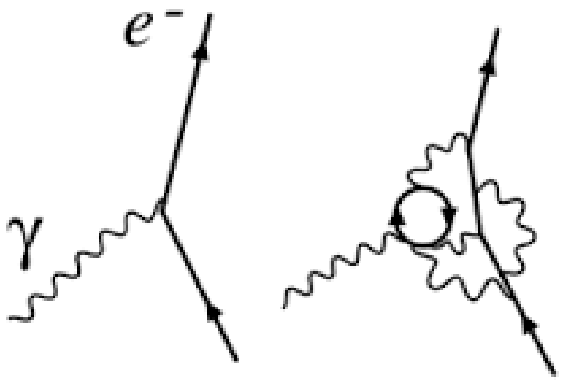
P. Pósfay, GGB, A. Jakovác: PASA **35** (2018) 19, PRC **97** (2018) 025803

Motivation for FRG

- It is hard to get effective action for an interacting field theory: e.g.: EoS for superdense cold matter ($T \to 0$ and finite μ)
- Taking into account quantum fluctuations using a scale, k
  - Classical action, $S = \Gamma_{k \to \Lambda}$ in the UV limit, $k \to \Lambda$
  - Quantum action, $\Gamma = \Gamma_{k \to 0}$ in the IR limit, $k \to 0$
- FRG Method
  - Smooth transition from macroscopic to microscopic
  - RG method for QFT
  - Non-perturbative description
  - Not depends on coupling
  - BUT: Technically it is NOT simple







Functional Renormalization Group (FRG)

- FRG is a general non-perturbative method to determine the effective action of a system.
- **Scale dependent effective action (k scale parameter)**

$$\partial_k \Gamma_k = \frac{1}{2} \int dp^D \; STr \left[\frac{\partial_k R_k}{\Gamma_k^{(2)} + R_k} \right]$$

Wetterich equation

k=Λ
Classical action

→ Integration →

k=0
Quantum fluctuations included

Ansatz: Interacting Fermi-gas model

Ansatz for the effective action:

$$\Gamma_k[\varphi,\psi] = \int d^4x \left[\bar{\psi}\,(i\partial\!\!\!/ - g\varphi)\,\psi + \frac{1}{2}(\partial_\mu\varphi)^2 - U_k(\varphi)\right]$$

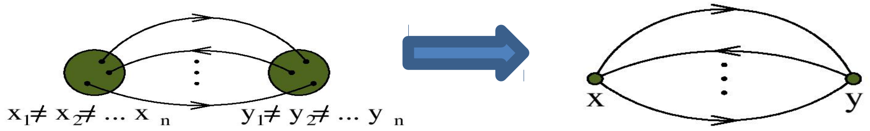
Fermions : *m=0*, **Yukawa-coupling** generates mass

Bosons: the **potential** contains self interaction terms

We study the scale dependence of the potential only!!

Interacting Fermi-gas at finite temperature

Ansatz for the effective action in LPA:



$$\Gamma_k\left[\varphi,\psi\right] = \int d^4x \left[\bar{\psi}\left(i\partial\!\!\!/ - g\varphi\right)\psi + \frac{1}{2}\left(\partial_\mu\varphi\right)^2 - U_k(\varphi)\right]$$

$$\Gamma_k\left[\psi\right] = \int d^4x \left[\frac{1}{2}\psi_i K_{k,ij}\psi_j + U_k\left(\psi\right)\right]$$

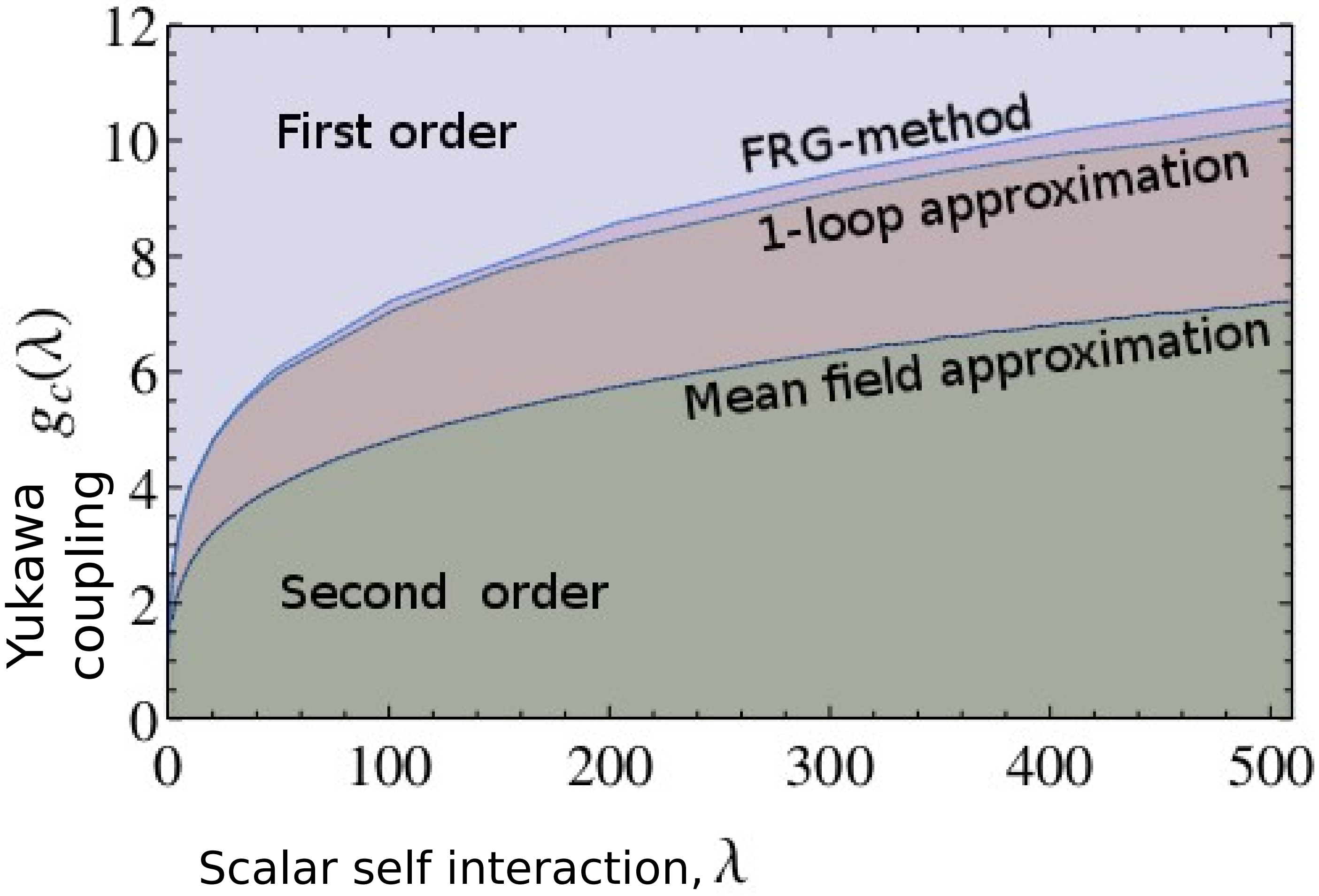
Wetterich-equation in LPA

$$\partial_k U_k = \frac{k^4}{12\pi^2}\left[\frac{1+2n_B(\omega_B)}{\omega_B} + 4\frac{-1+n_F(\omega_F-\mu)+n_F(\omega_F+\mu)}{\omega_F}\right]$$

Bosonic part **Fermionic part**

$$U_\Lambda(\varphi) = \frac{m_0^2}{2}\varphi^2 + \frac{\lambda_0}{24}\varphi^4 \qquad \omega_F^2 = k^2 + g^2\varphi^2 \qquad \omega_B^2 = k^2 + \partial_\varphi^2 U \qquad n_{B/F}(\omega) = \frac{1}{1 \mp e^{-\beta\omega}}$$

Result: Phase structure of interacting Fermi gas model



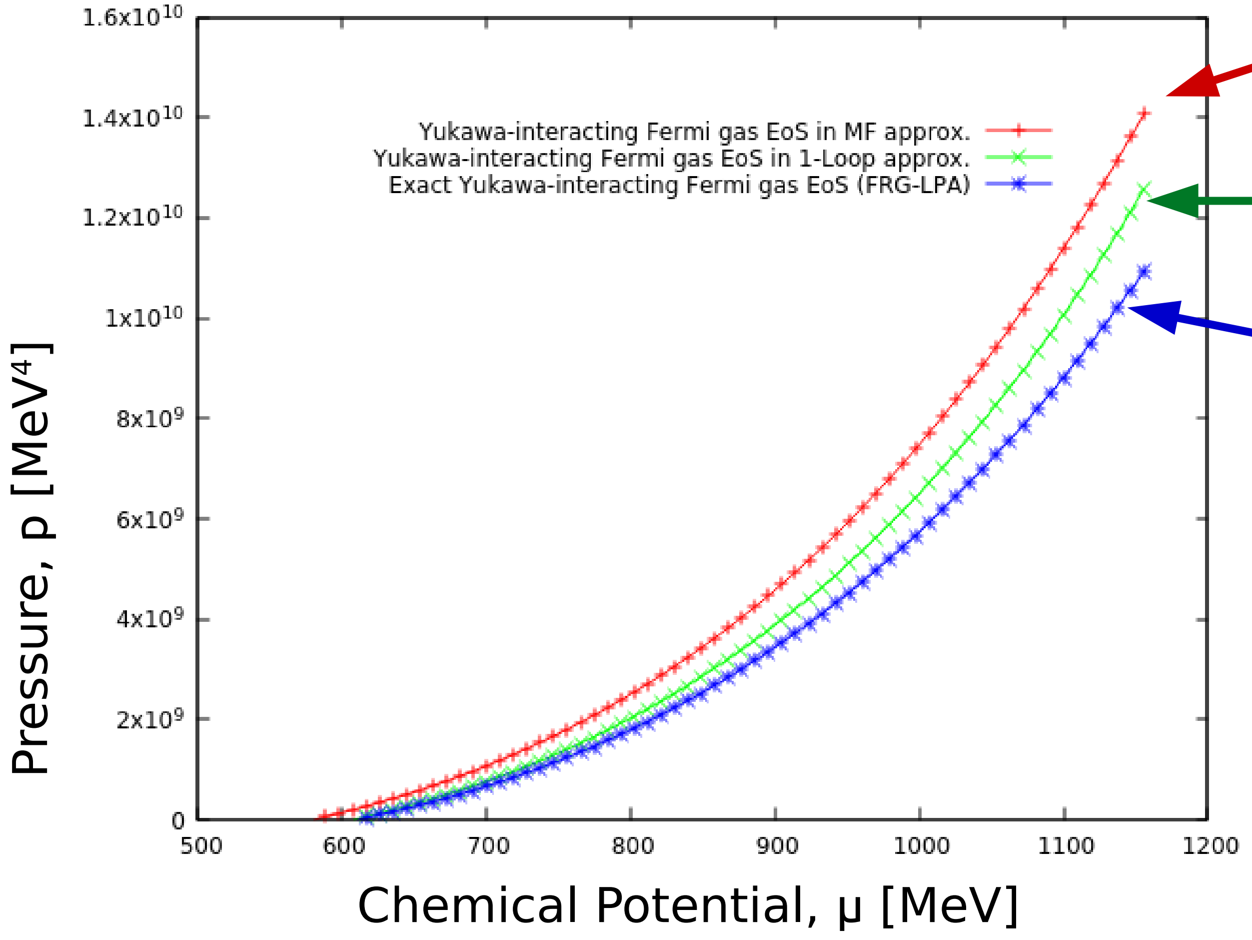
Exact FRG solution counts all quantum fluctuations

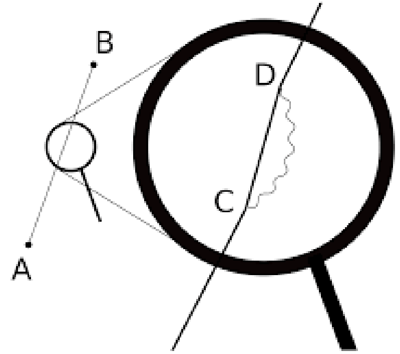
1-Loop approximation has only tree diagrams

Mean Filed solution contains averaged effect of interactions

In the phase structure, FRG and 1L are very similar if the LO has the strongest contribution.

Result: Comparison of MF, 1L, & FRG-based EoS





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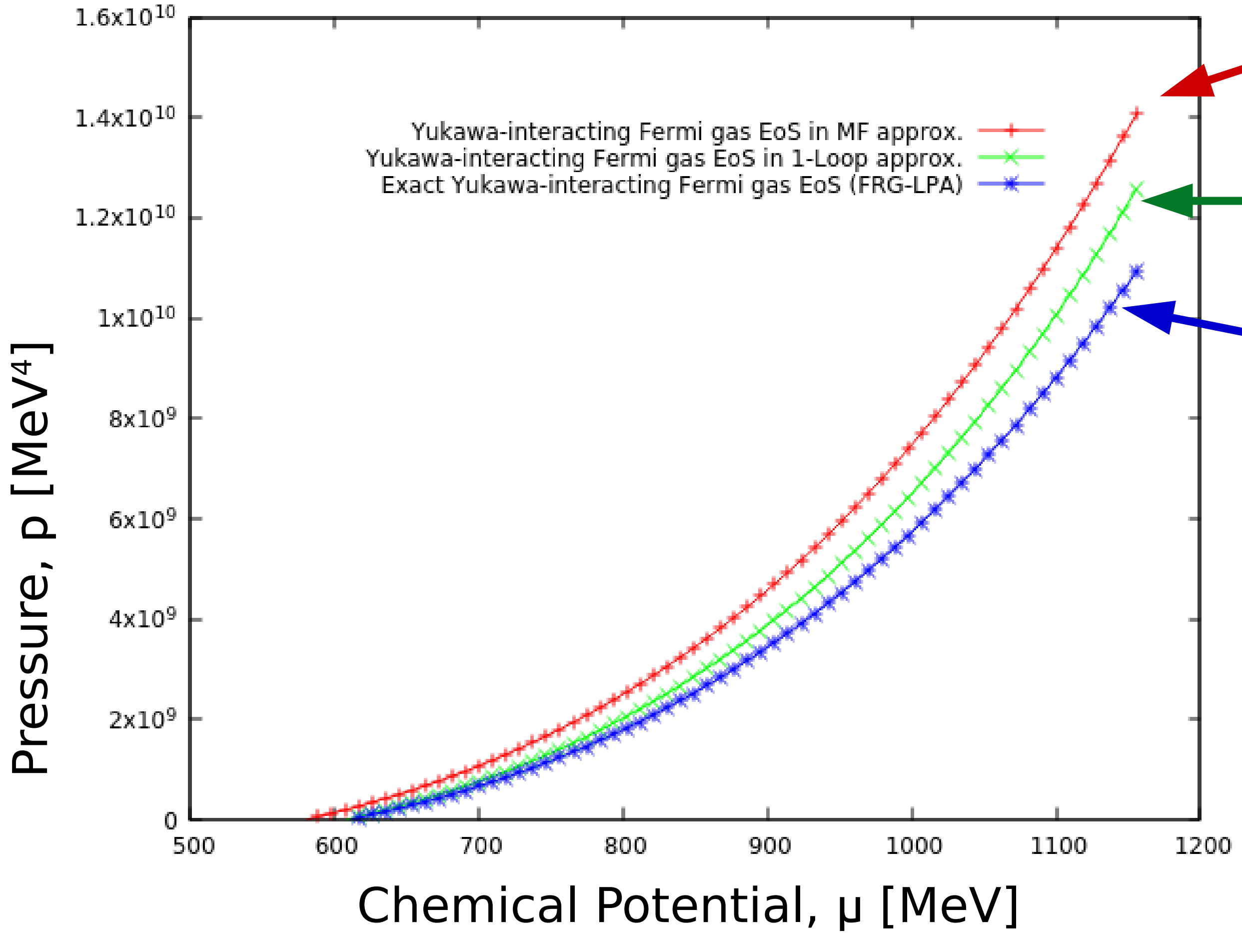
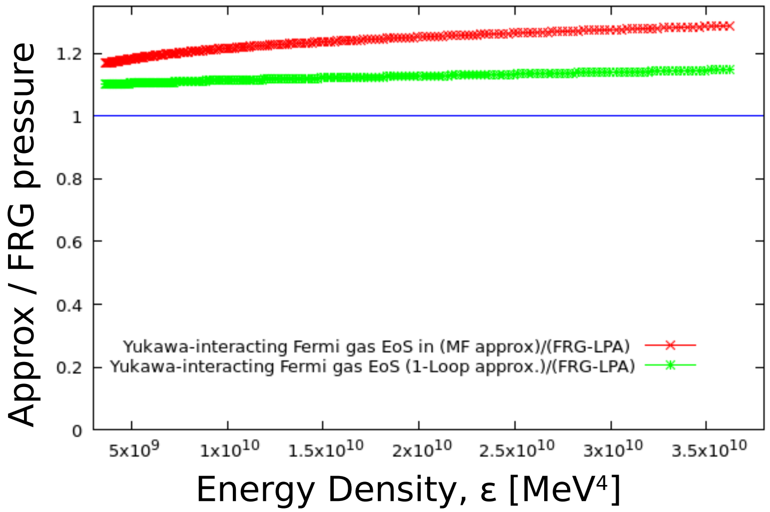


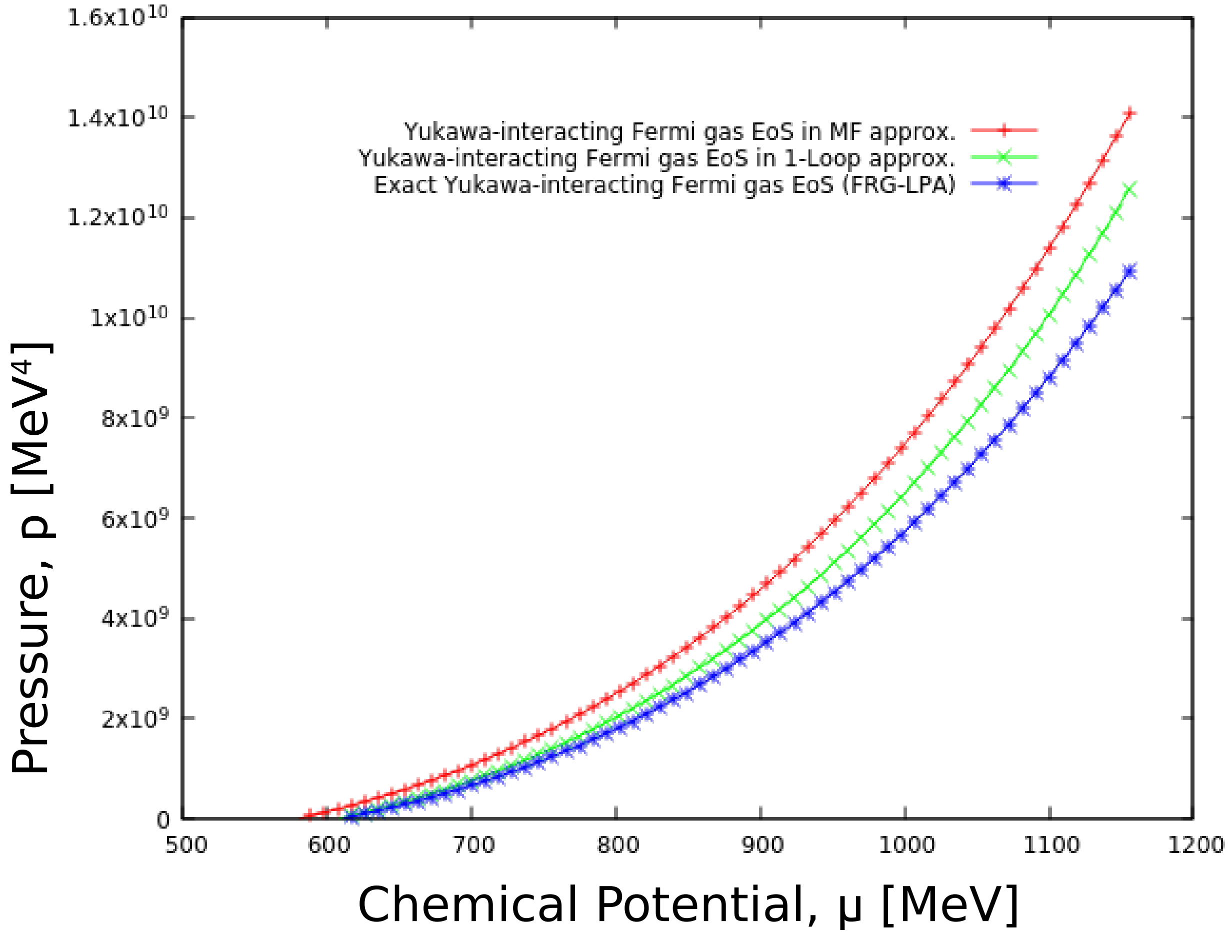
MF is 25% stiffer than the FRG

1L is 10% stiffer than the FRG

Exact FRG solution softest

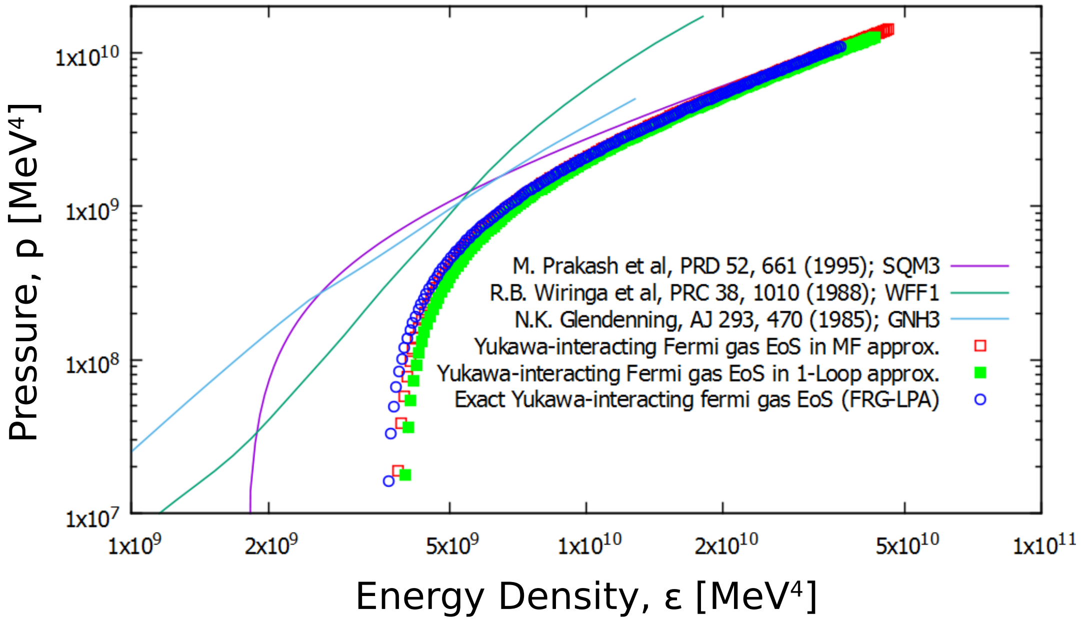


Result: Comparison to other EoS models

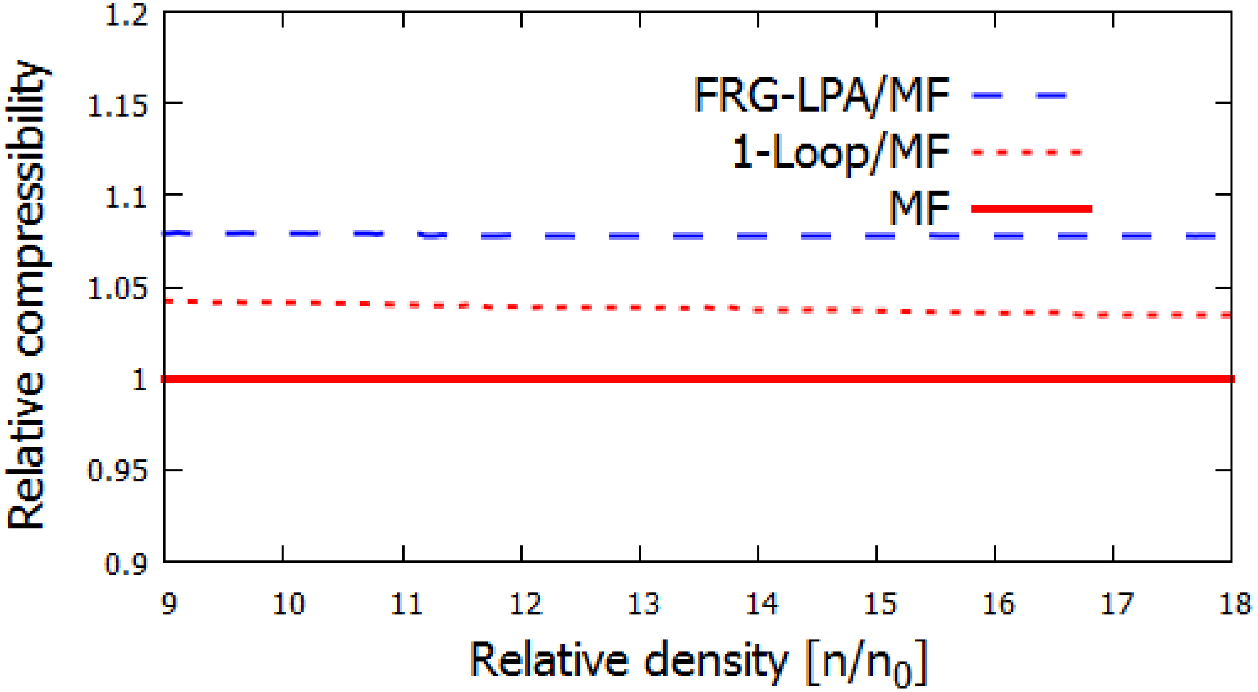


Compare FRG to SQM3, GNH3, WFF1

- Overlap with SQM3 at high ε
- Cutoff, ε_{Cut} is also higher
- Approximations differ slightly



Result: Comparison of compressibility in the models



Compare FRG to 1L and MF

- Compressibility:

$$\frac{1}{\chi} = n\frac{\partial P}{\partial n} = 2n^2\frac{\partial}{\partial n}(E/A) + n^3\frac{\partial^2}{\partial n^2}(E/A)$$

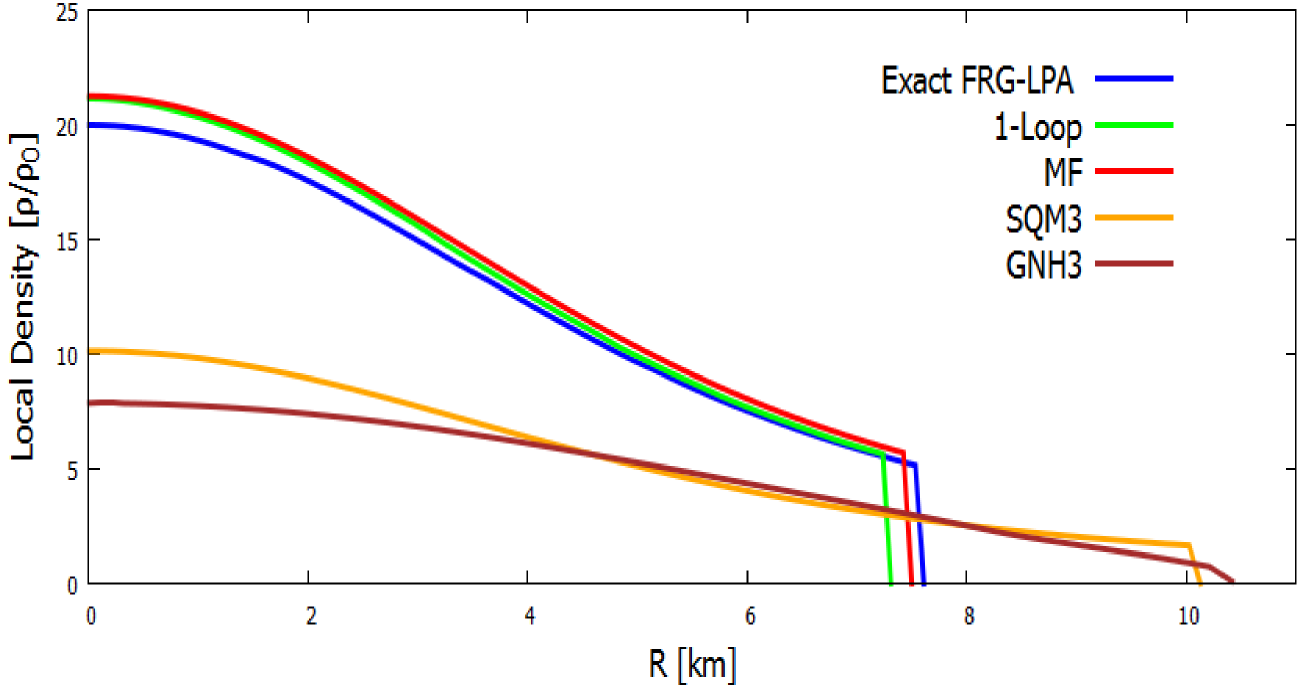
- Compression modulus

$$K = k_F^2\frac{\partial^2}{\partial k_F^2}(E/A) = \frac{9}{n_0\,\chi}$$

- The difference between the models is about ~10%

Result: Test in a Compact Star

- Compare FRG EoS to SQM3, GNH3 → TOV result: density function



Compare FRG to 1L and MF

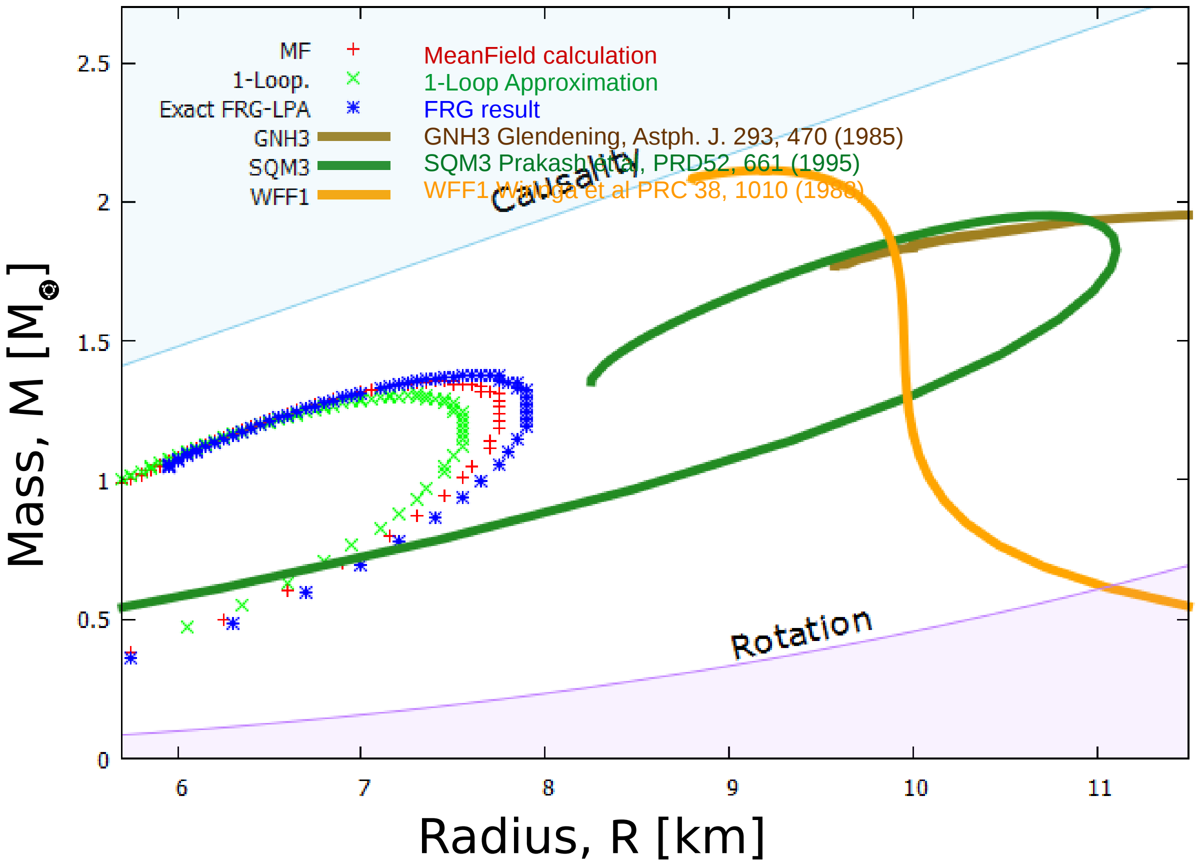
- Soft FRG make biggest star
- High-ε part is similar for all
- Difference: ~5% (.1 $M_\odot$ and .5 km)

FRG to SQM3, GNH3

- FRG: small stars 1.4$M_\odot$ and 8 km
- Other models: larger radii and less central density

Result: Test in a Compact Star

- Compare FRG EoS to SQM3, GNH3, WFF1 → TOV result on M(R) diagram



Compare FRG to 1L and MF

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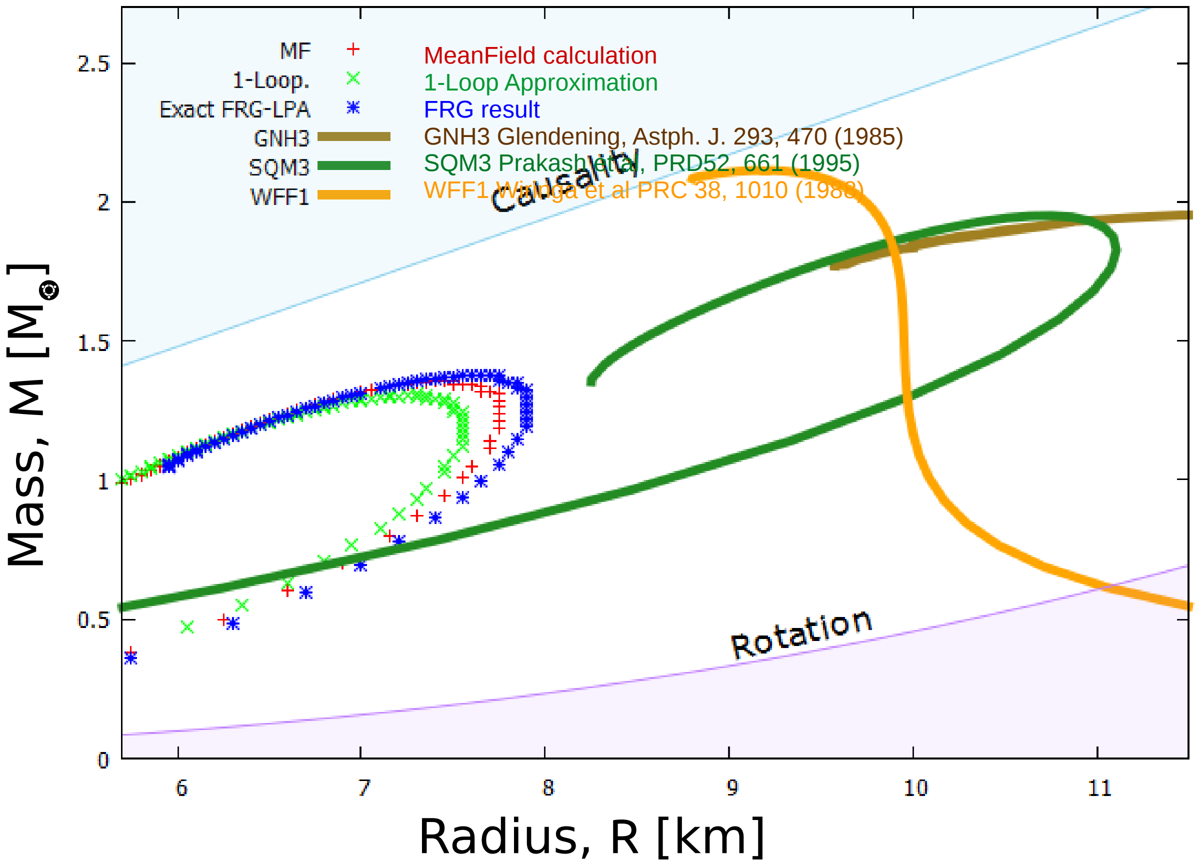
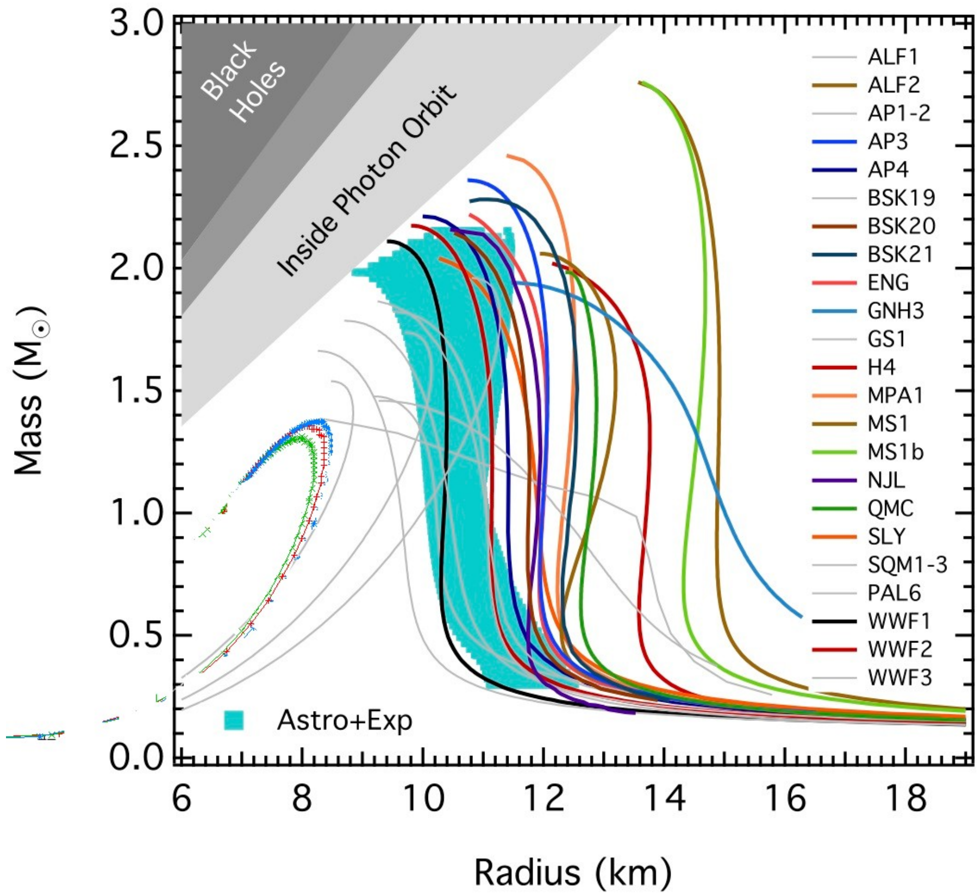
FRG to SQM3, GNH3, WFF1

- Small stars 1.4 $M_\odot$ and 8 km
- Overlap with SQM3 at high ε
- Interaction (ω) will increase

Result: Test in a Compact Star

- Compare FRG EoS to SQM3, GNH3, WFF1 → TOV result on M(R) diagram





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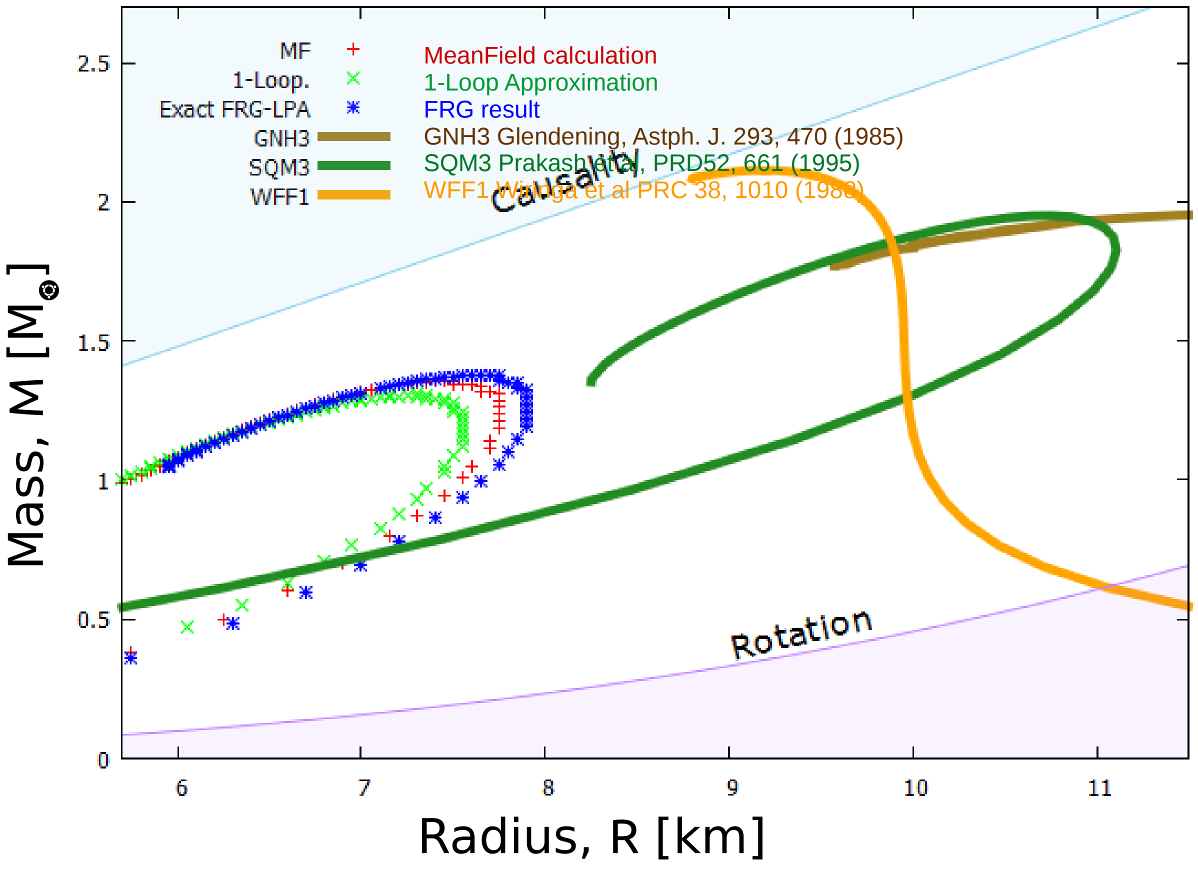
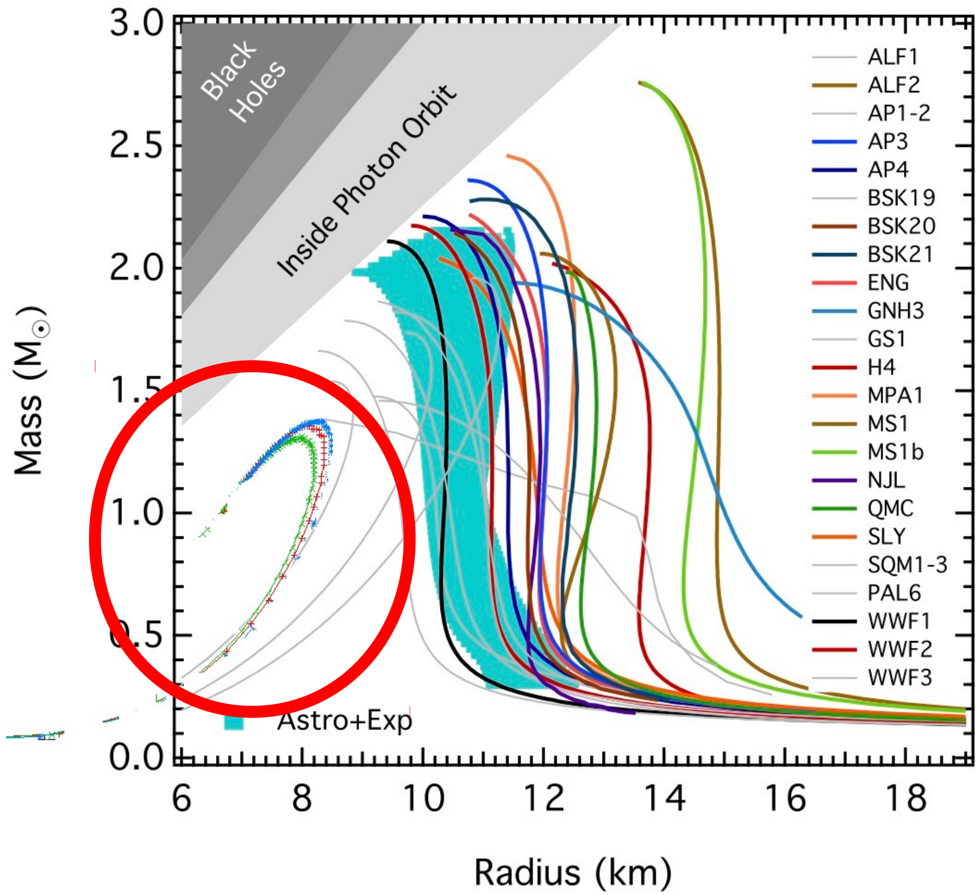
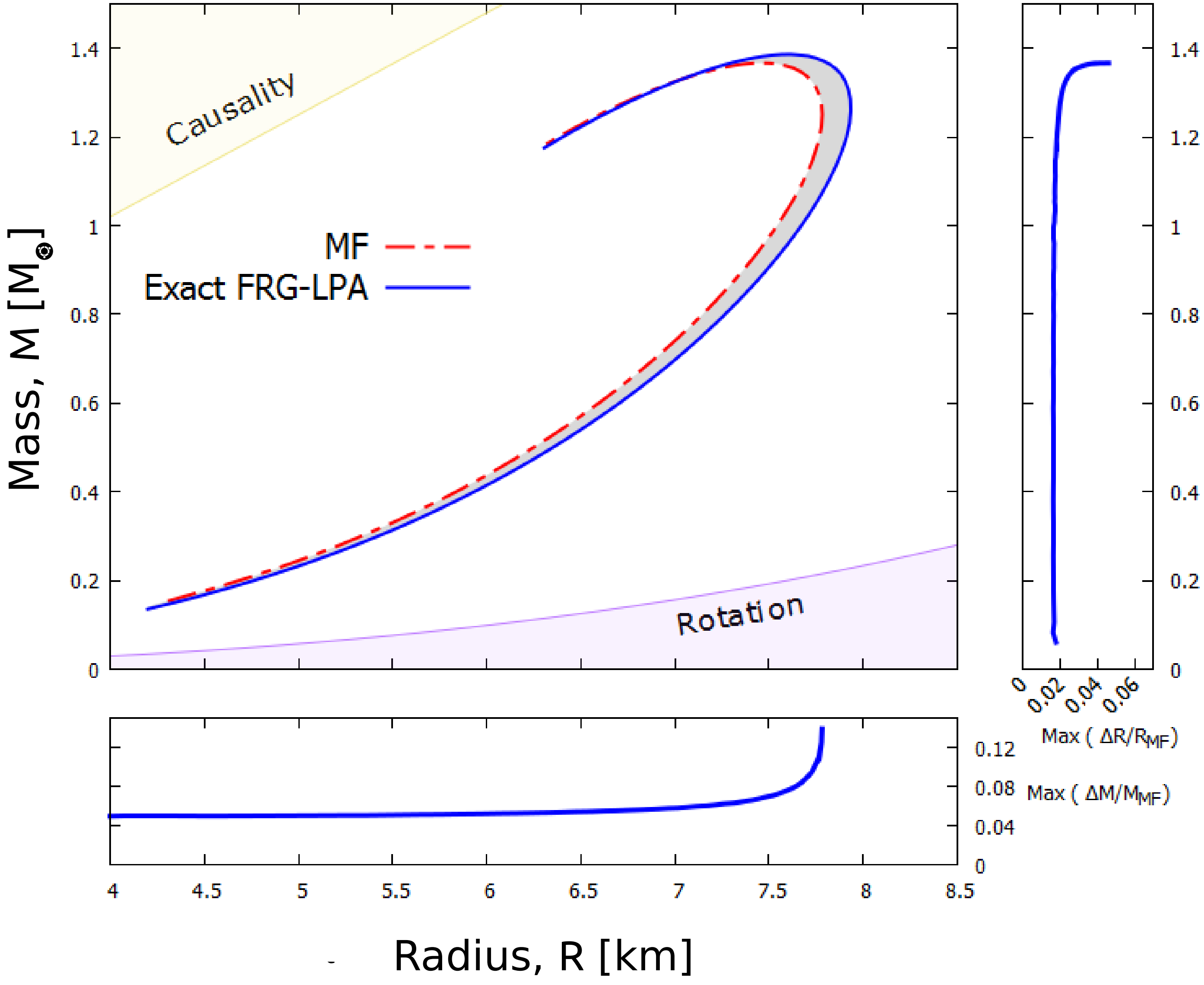




Test: Can we test this by observations?

- Compare different EoS results on M(R) diagram: MF & FRG
- Maximal relative differences are also plotted



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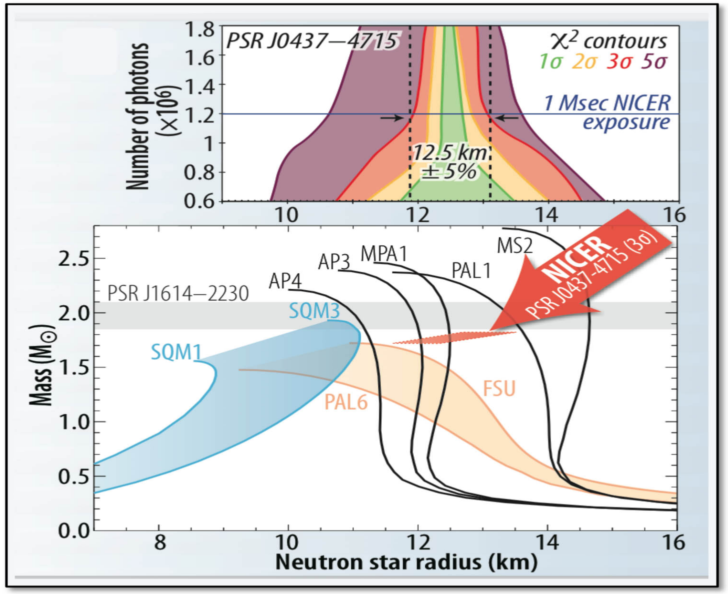

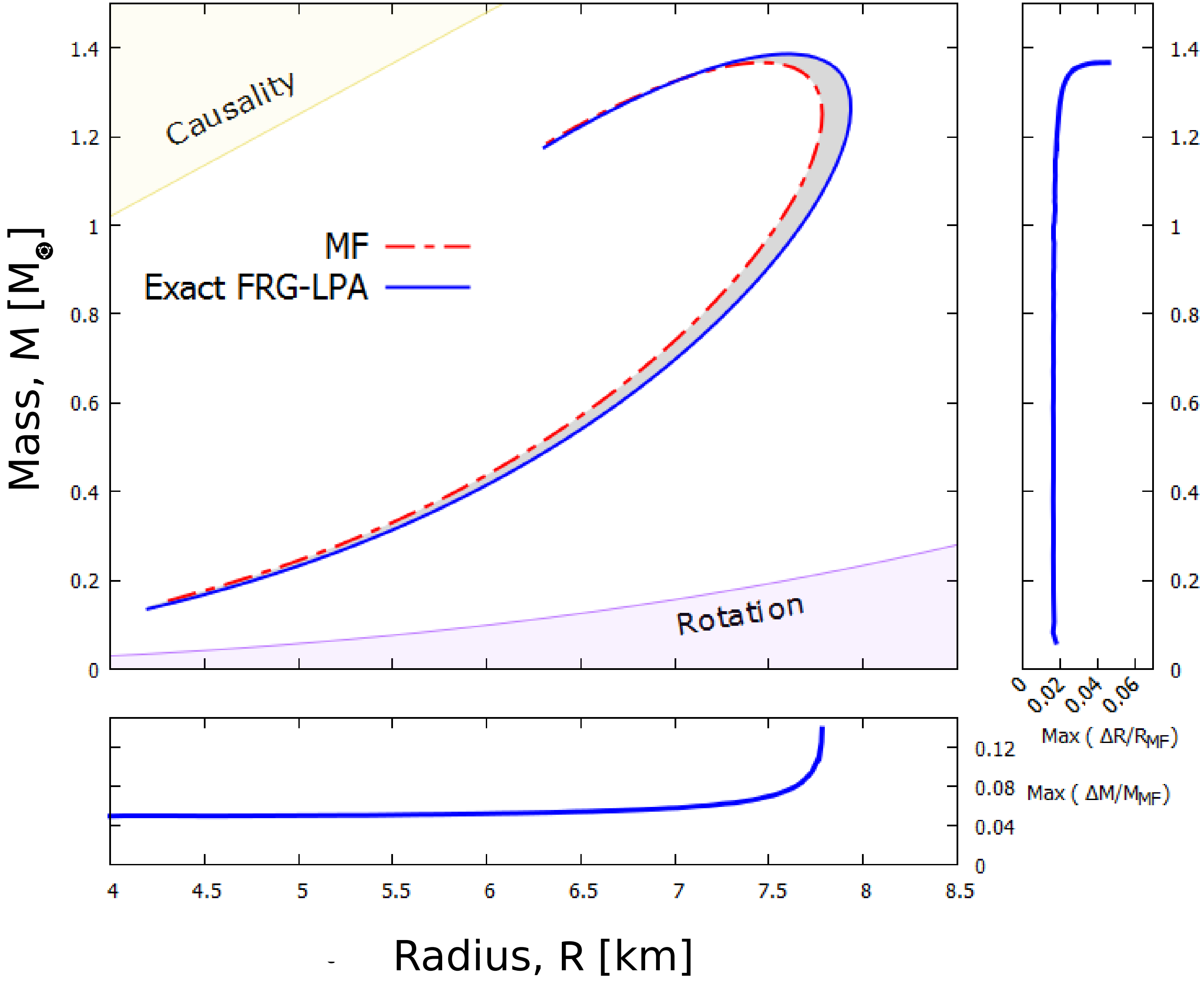

The summary of the theoretical uncertainties

- The magnitude of the uncertainties of (astro)physical observables
  - Microscopical observables are maximum: 10-25%
  - Macroscopical astrophysical ones are maximum: 5-10%
  - Measurement resolution limit is about: 10%

Observable	Max theory uncertainty (%)
Potential, U(φ)	< 25%
Phase diagram (g_c)	< 25%
EoS p(μ),p(ε)	< 25%
Compressibility	< 10%
ε(R)	~ 5%
M(R) diagram	< 10% (M) < 5% (R)
Compactness	< 10% (M) < 5% (R)

2) Uncertainties from the parameters of realistic nuclear matter

P. Pósfay, GGB, A. Jakovác: arXiv:1905.01872 [hep-th] (symmetric case)

Modified σ-ω model in mean field

$$\mathcal{L}_{MF} = \sum_{i=1,2} \bar{\psi}_i \left(i\partial\!\!\!/ - m_N + g_\sigma \overline{\sigma} - g_\omega \gamma^0 \, \overline{\omega}_0 \right) \psi_i$$

Nucleon effective mass ($- m_N + g_\sigma \overline{\sigma}$)

Proton and neutron

$$- \frac{1}{2} m_\sigma^2 \overline{\sigma}^2 - \lambda_3 \, \overline{\sigma}^3 - \lambda_4 \, \overline{\sigma}^4$$

Scalar meson self interaction terms

$$+ \frac{1}{2} m_\omega^2 \, \overline{\omega}_0^2$$

Vector meson

Extra terms

$$+ \frac{1}{2} m_\rho^2 \, \rho_\mu^a \, \rho^{\mu\, a}$$

Tensor meson

$$+ \overline{\Psi}_e \left(i\partial\!\!\!/ - m_e \right) \Psi_e$$

Electron in β-equilibrium

$$\mu_n = \mu_p + \mu_e$$

Modified σ-ω model in mean field

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Scalar meson self interaction terms

$$+\frac{1}{2} m_\omega^2 \overline{\omega}_0^2$$

Vector meson

Extra terms

p – n Nuclear force

$$+\frac{1}{2} m_\rho^2 \rho_\mu^a \rho^{\mu\, a}$$

Tensor meson

$$+\overline{\Psi}_e \left(i\partial\!\!\!/ - m_e \right) \Psi_e$$

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Proton and neutron

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Scalar meson self interaction terms

$$+ \frac{1}{2} m_\omega^2 \overline{\omega}_0^2$$

Vector meson

$$+ \frac{1}{2} m_\rho^2 \rho_\mu^a \rho^{\mu\, a}$$

Isospin asymmetry

Tensor meson

$$+ \overline{\Psi}_e \left(i\partial\!\!\!/ - m_e \right) \Psi_e$$

Electron in β-equilibrium

$$\mu_n = \mu_p + \mu_e$$

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Proton and neutron

Nucleon effective mass

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Vector meson

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Tensor meson

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$$\mu_n = \mu_p + \mu_e$$

Modified σ-ω model in mean field

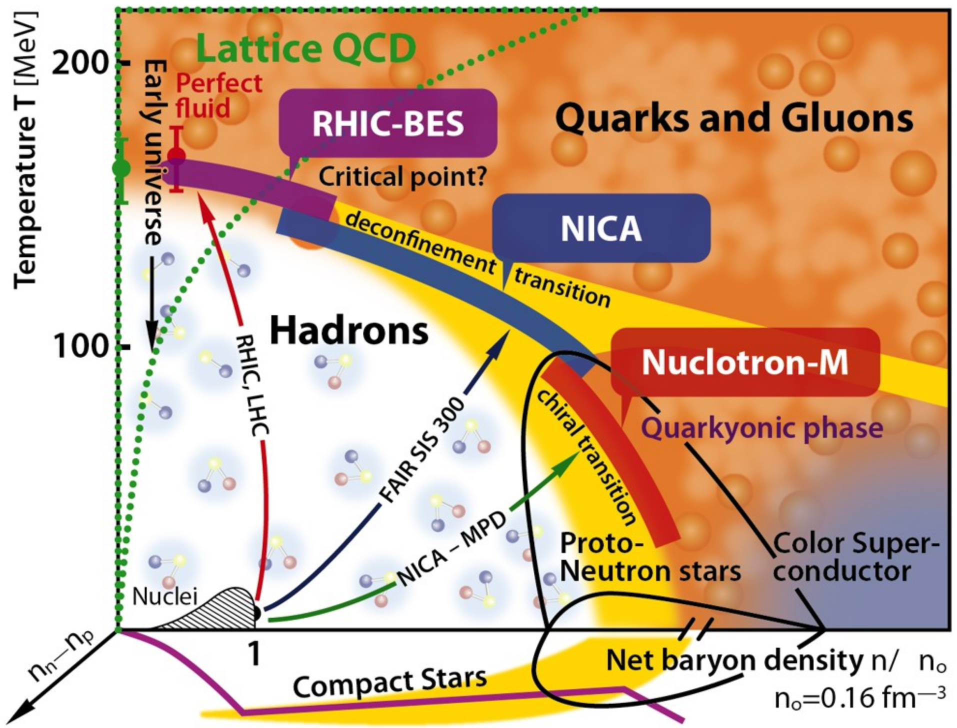
- **Theoretical mean field model:**
  - Symmetric case: 3 combinations with the higher-order scalar meson self-interaction terms to original Walecka:
  - Asymmetric case: tensor force is added to the interaction in addition to the electrons, for β-equilibrium: $\mu_n = \mu_p + \mu_e$
- **Parameters of the theoretical model**
  - Fit couplings/masses/etc. according to the Rhoades–Ruffini theorem in agreement with experimental data.
  - Parameters are usually non-independent: optimalization of the parameters need to perform → similar EoS
- **Cross check the consistency with the the existing EM, GR, HIC, etc data + errors → Theoretical uncertainties**

$$- \lambda_3 \, \overline{\sigma}^3 - \lambda_4 \, \overline{\sigma}^4$$

$$+ \frac{1}{2} m_\rho^2 \, \rho_\mu^a \, \rho^{\mu\, a}$$

$$+ \overline{\Psi}_e \, (i\not{\partial} - m_e) \, \Psi_e$$

Parameters to fit normal nuclear matter



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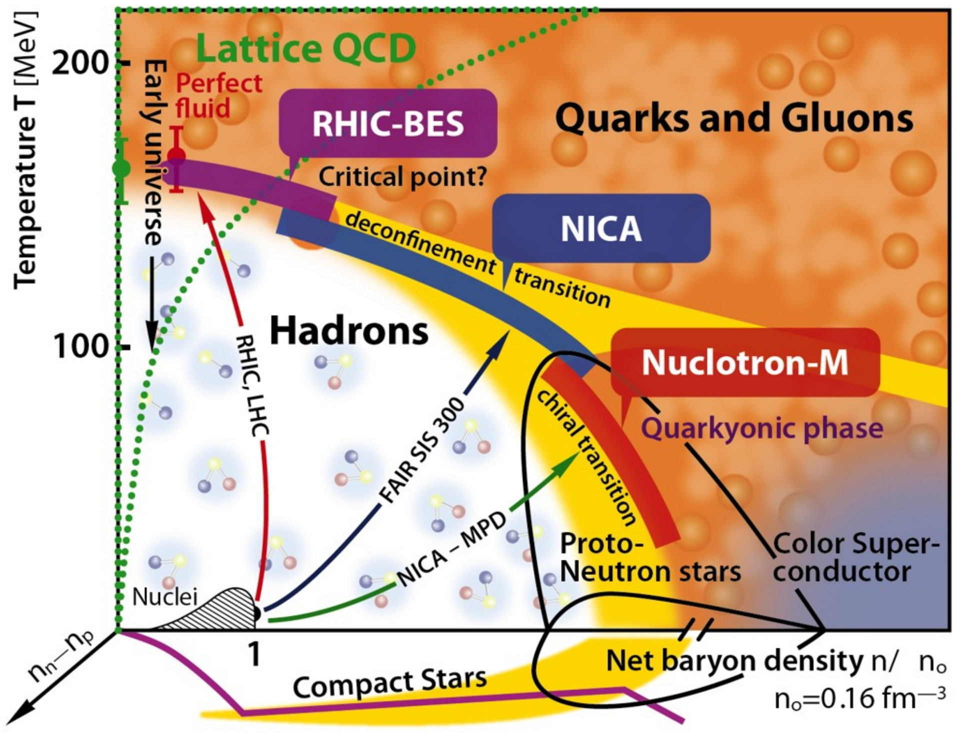
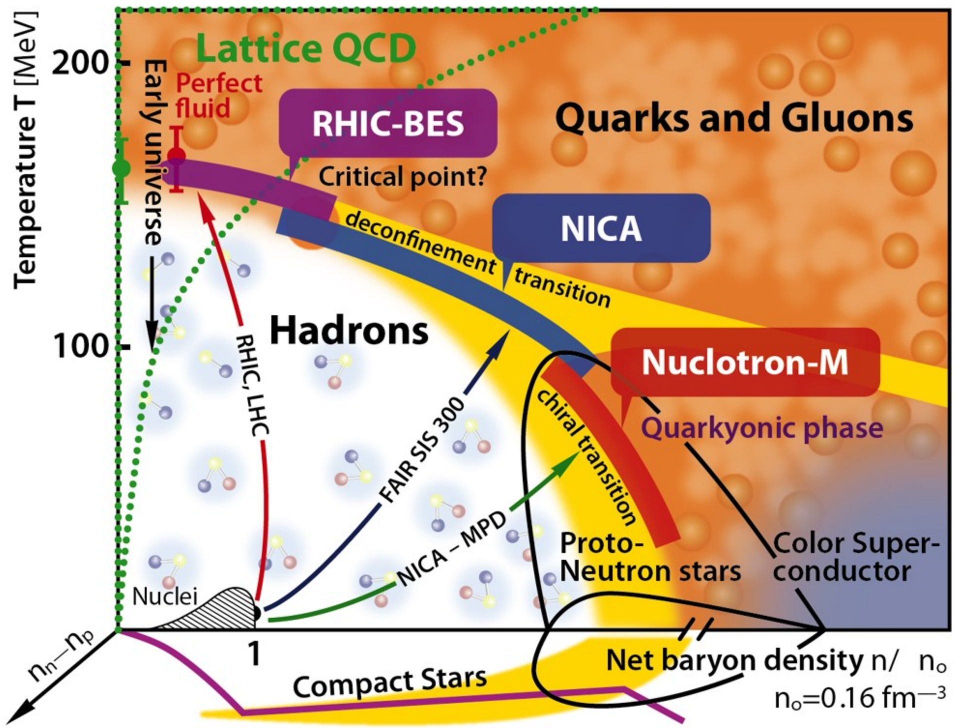


Parameter	Value
Saturation density	0.156 $1/fm^3$
Binding energy	-16.3 MeV
Nucleon effective mass	0.6 m_N
Nucleon Landau mass	0.83 m_N
incompressibility	240 MeV
Asymmetry energy	32.5 MeV

Parameters to fit normal nuclear matter



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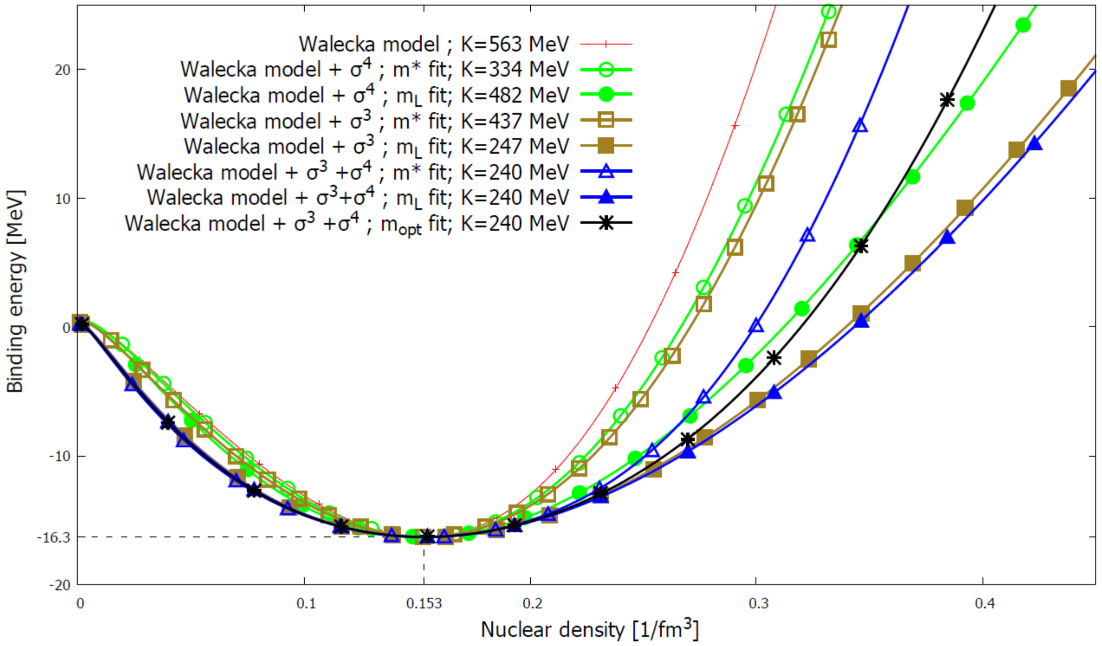
Incompressibility $$K = k_F^2 \frac{\partial^2(\epsilon/n)}{\partial k_F^2} = 9\,\frac{\partial p}{\partial n}$$

Landau mass $$m_L = \frac{k_F}{v_F} = \sqrt{k_F^2 + m_{N,eff}^2}$$

Parameters to fit normal nuclear matter



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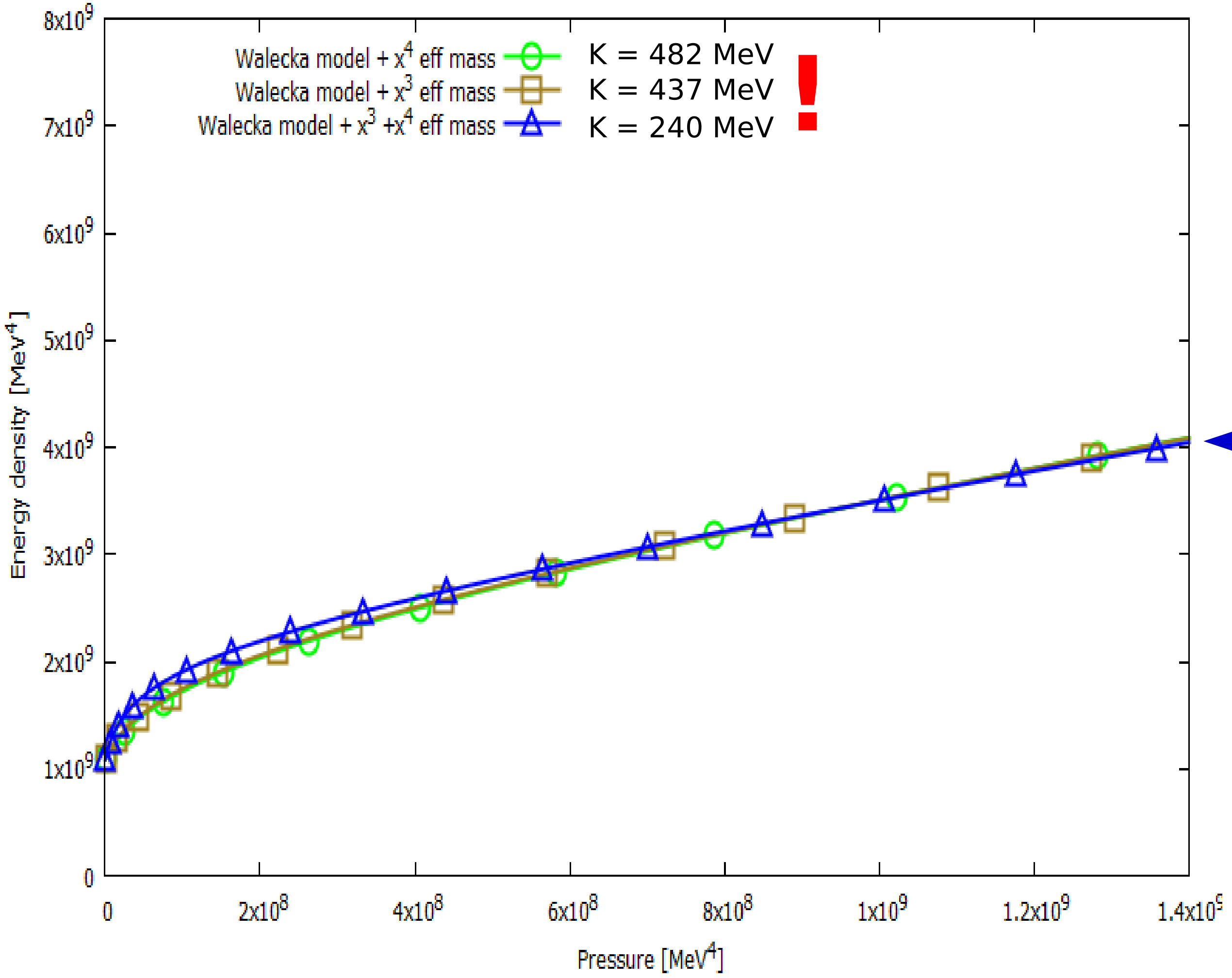
Incompressibility $$K = k_F^2 \frac{\partial^2(\epsilon/n)}{\partial k_F^2} = 9\frac{\partial p}{\partial n}$$

The effective mass and Landau mass
are NOT independent!
The can not be fitted simultaneously

Landau mass $$m_L = \frac{k_F}{v_F} = \sqrt{k_F^2 + m_{N,eff}^2}$$

The Equation of State of different model fits



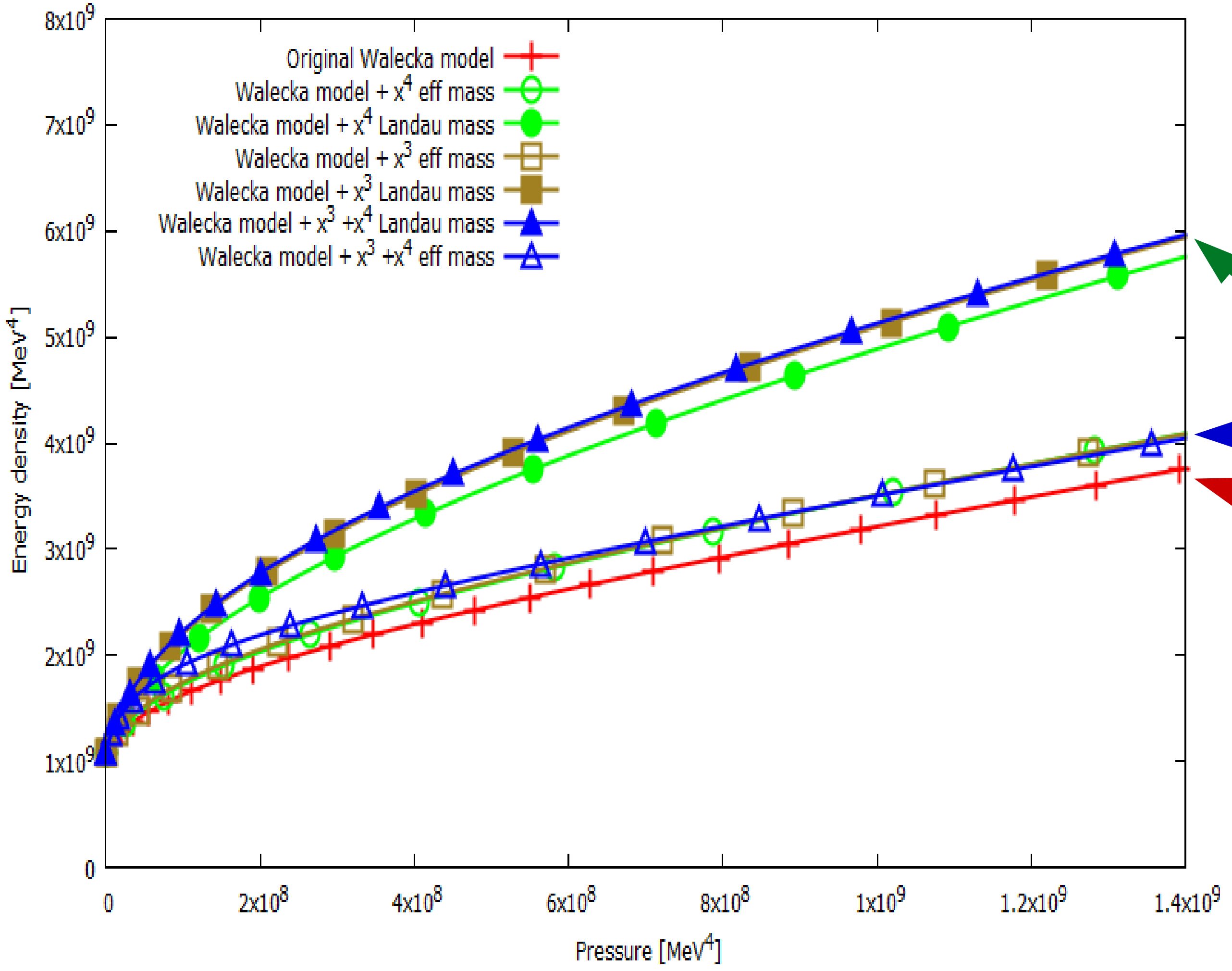
Adding higher-order terms

- helps, at lower pressure
- more parameter more constraints:

Effective mass fit $m_{Eff} = 0.6\ m_N$

Different models give similar EoS

The Equation of State of different model fits



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- more parameter more constraints:

Landau mass fit $m_{Eff} = 0.83\ m_N$

Effective mass fit $m_{Eff} = 0.6\ m_N$

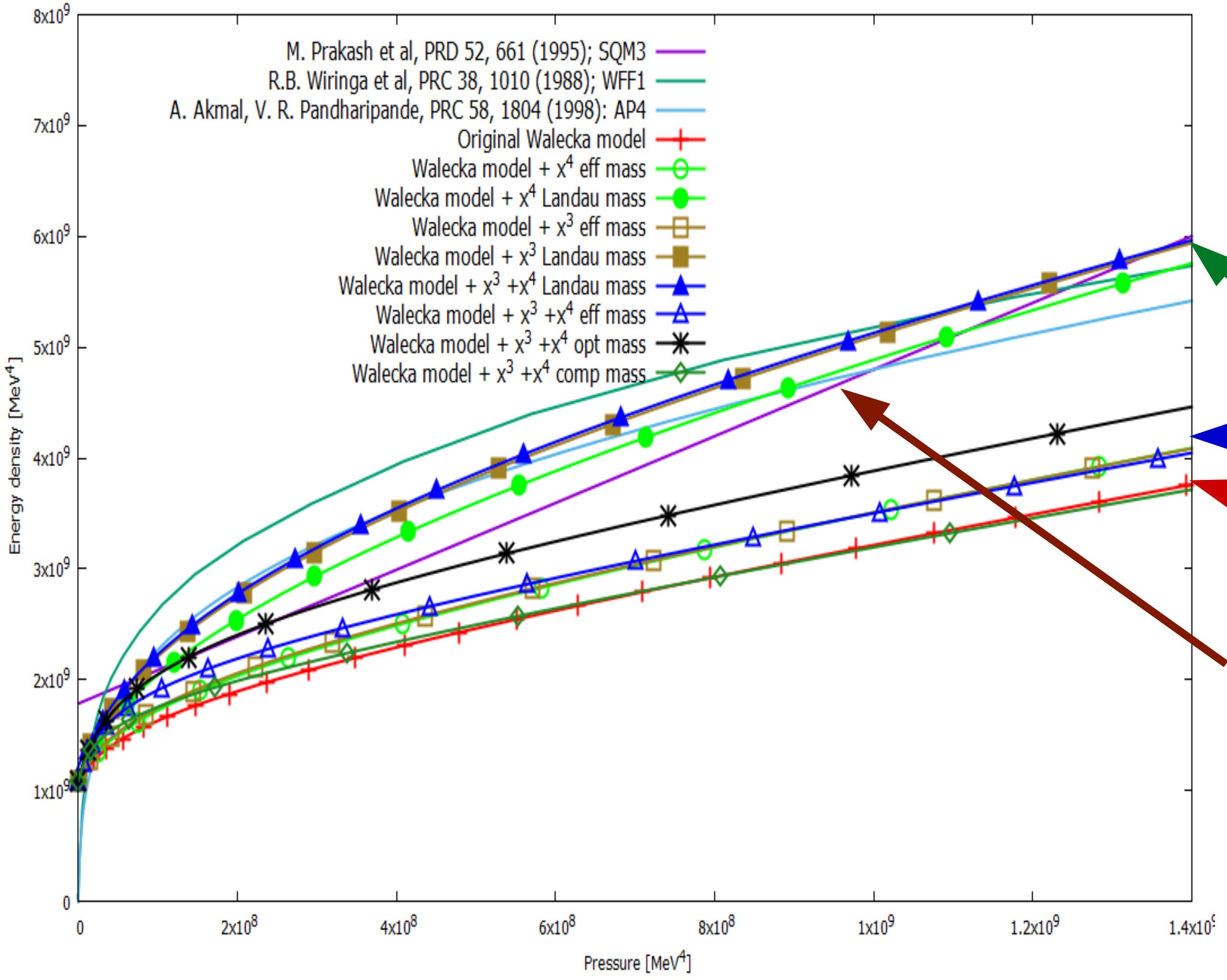
Original Walecka model

Different models give similar EoS

Depending on the ‘fit-type’

→ bands appear

The Equation of State of different model fits



Adding higher-order terms

- helps, at lower pressure
- more parameter more constraints:

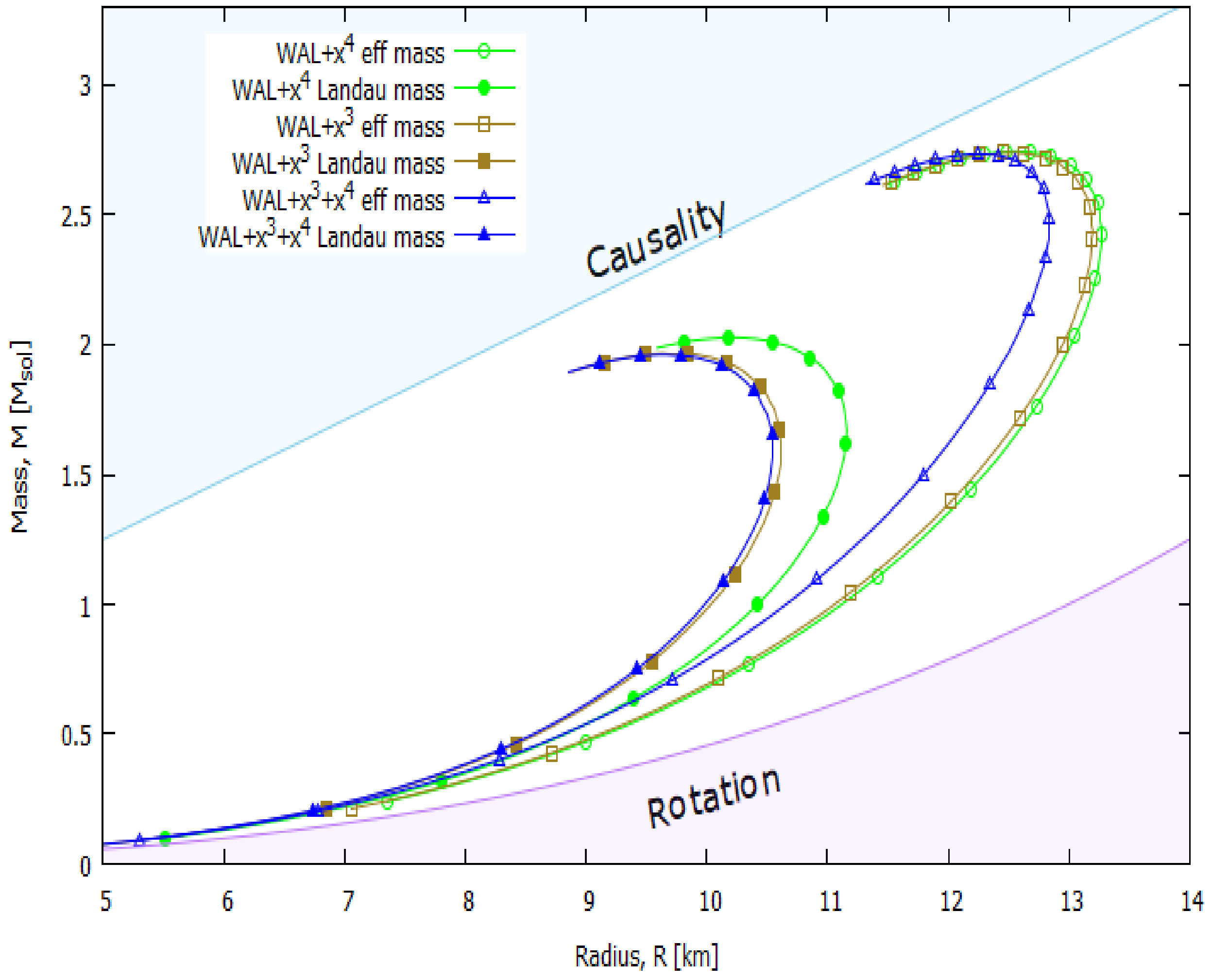
Landau mass fit $m_{Eff} = 0.83\ m_N$

Effective mass fit $m_{Eff} = 0.6\ m_N$

Original Walecka model

Realistic nuclear matter EoSs, like WFF1, AP4 (SQM) support the Landau mass fits well.

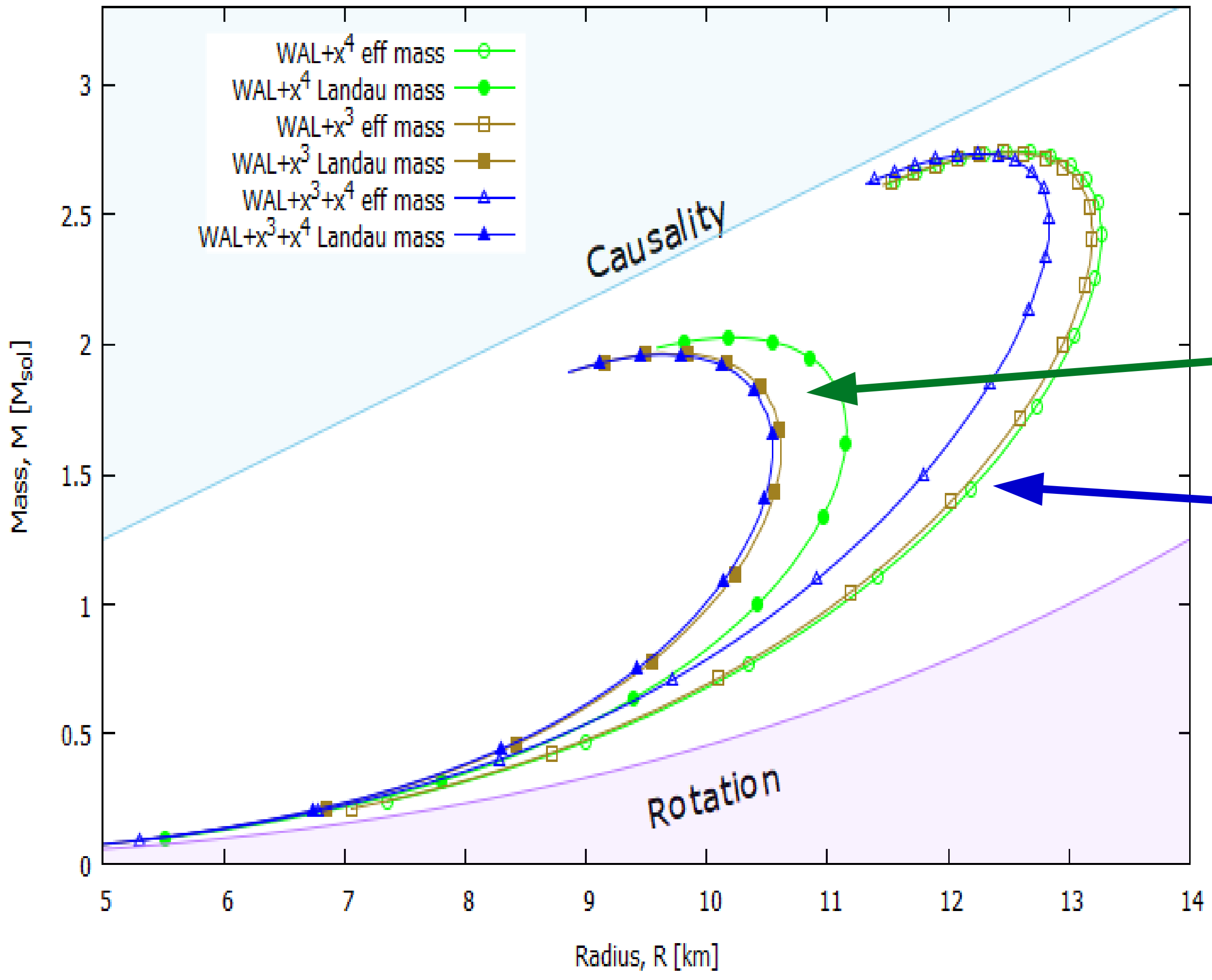
The M-R diagrams: EoS & effective mass fit



SYMMETRIC nuclear matter EoS

- Cases with extra x^3 and/or x^4 terms provide similar band structures in the M-R diagram

The M-R diagrams: EoS & effective mass fit



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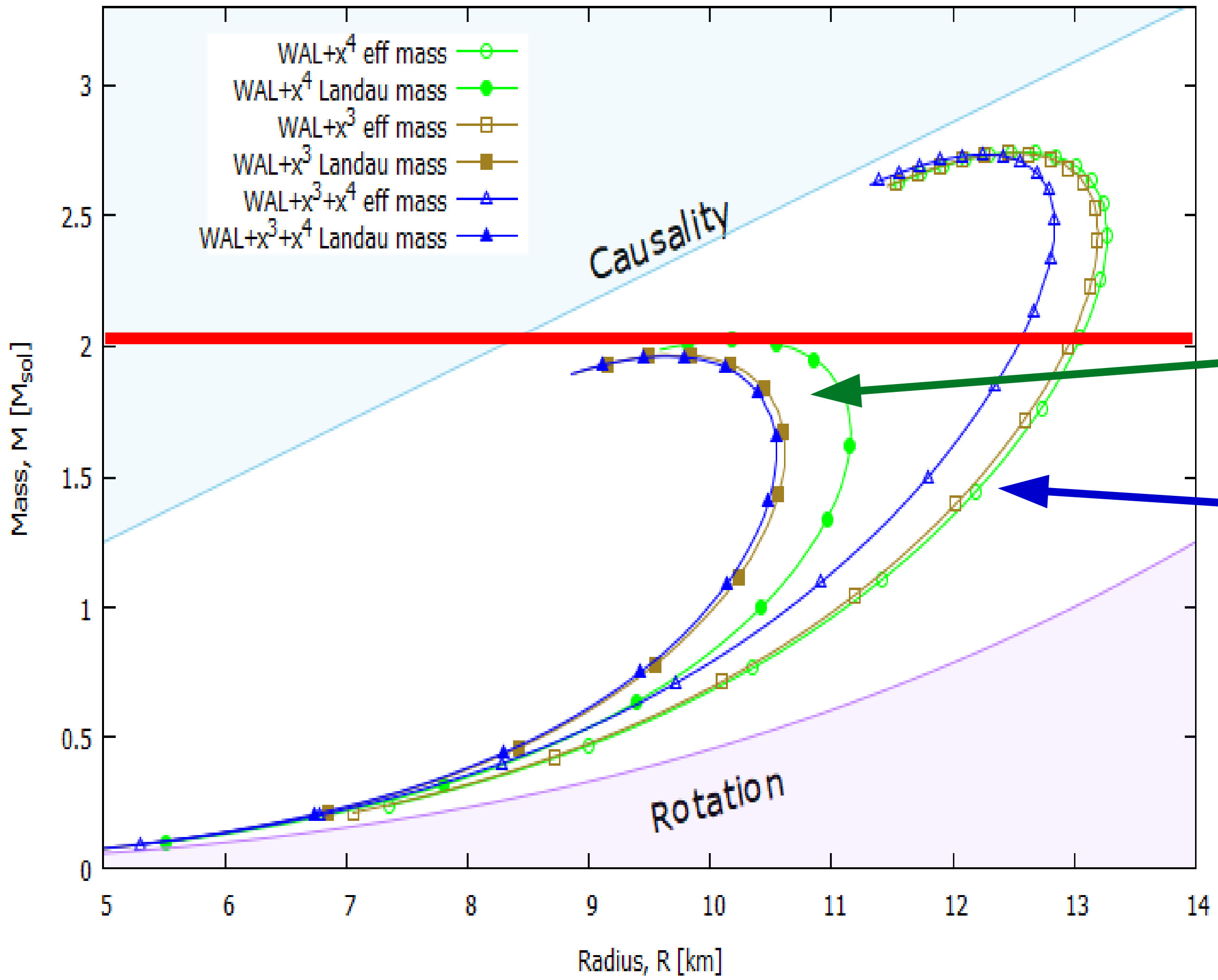


SYMMETRIC nuclear matter EoS

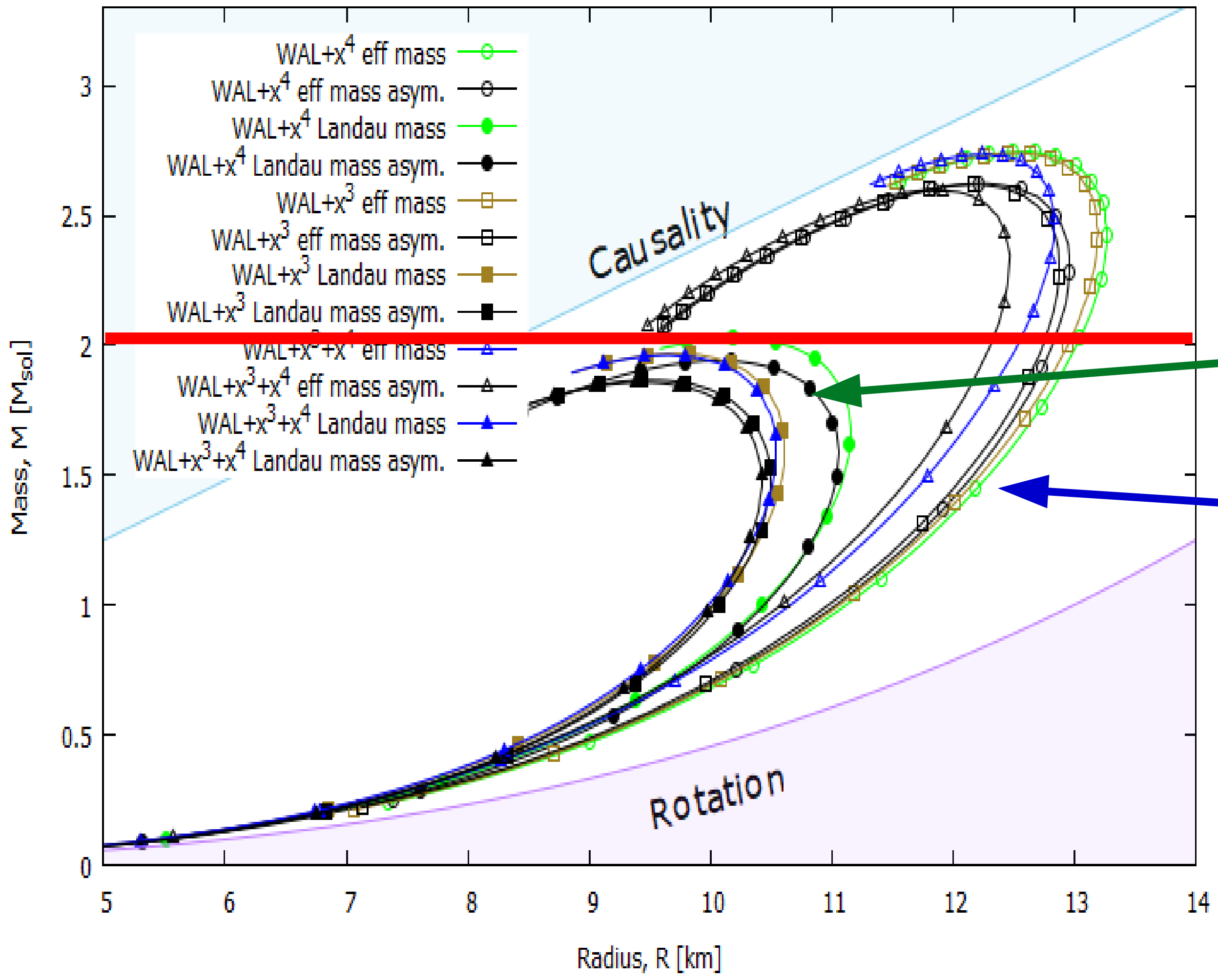
- Cases with extra x^3 and/or x^4 terms provide similar band structures in the M-R diagram

Landau mass fit $m_{Eff} = 0.83\ m_N$

Effective mass fit $m_{Eff} = 0.6\ m_N$

→ Landau mass fits provide compact star with lower M_{max} but closer to the observations

The M-R diagrams: EoS & effective mass fit



ASYMMETRIC nuclear matter EoS

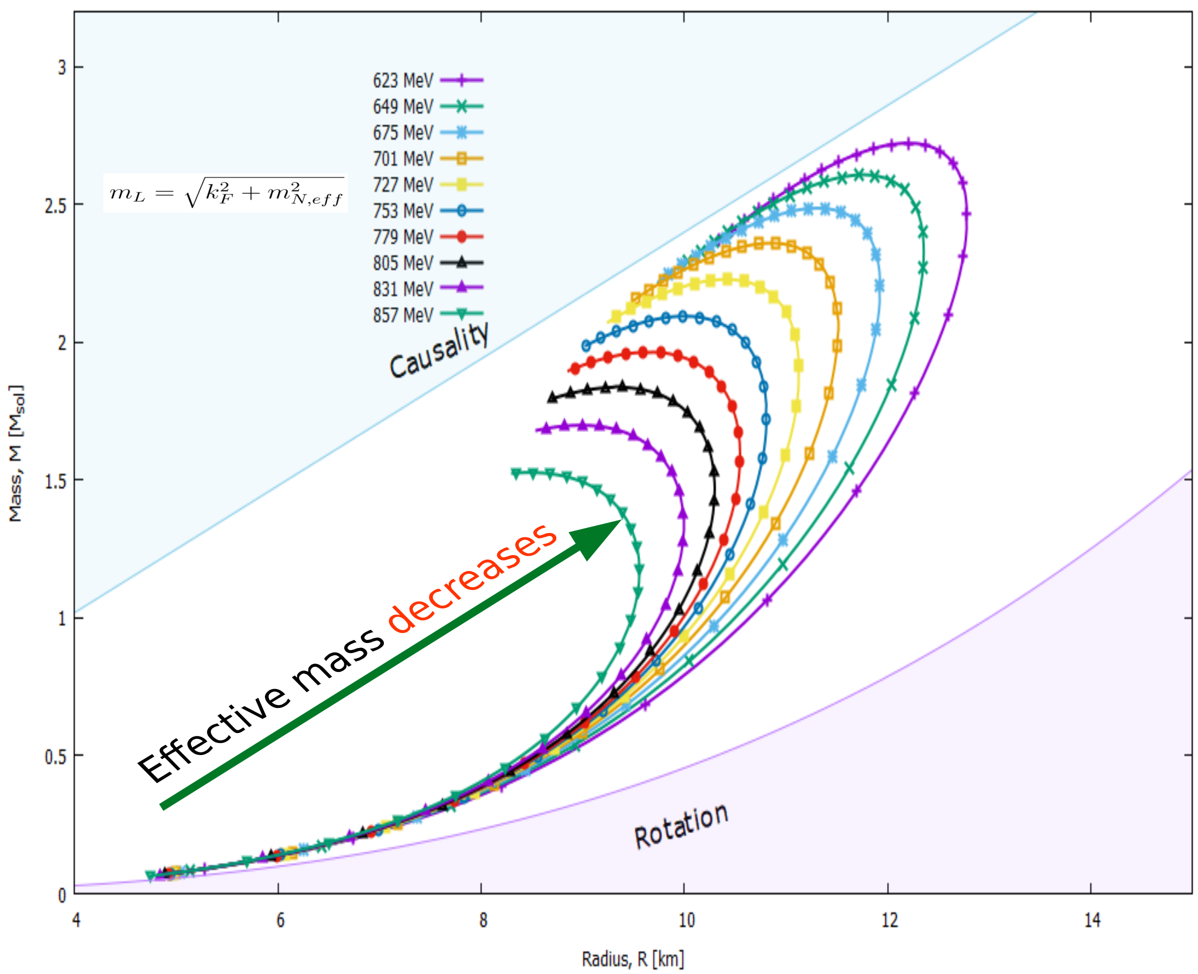
- Cases with extra x^3 and/or x^4 terms provide similar band structures in the M-R diagram

Landau mass fit $m_{Eff} = 0.83\ m_N$

Effective mass fit $m_{Eff} = 0.6\ m_N$

→ Nuclear ASYMMETRY has weak decreasing effect on the M_{max}

The M-R diagrams: EoS & effective mass fit

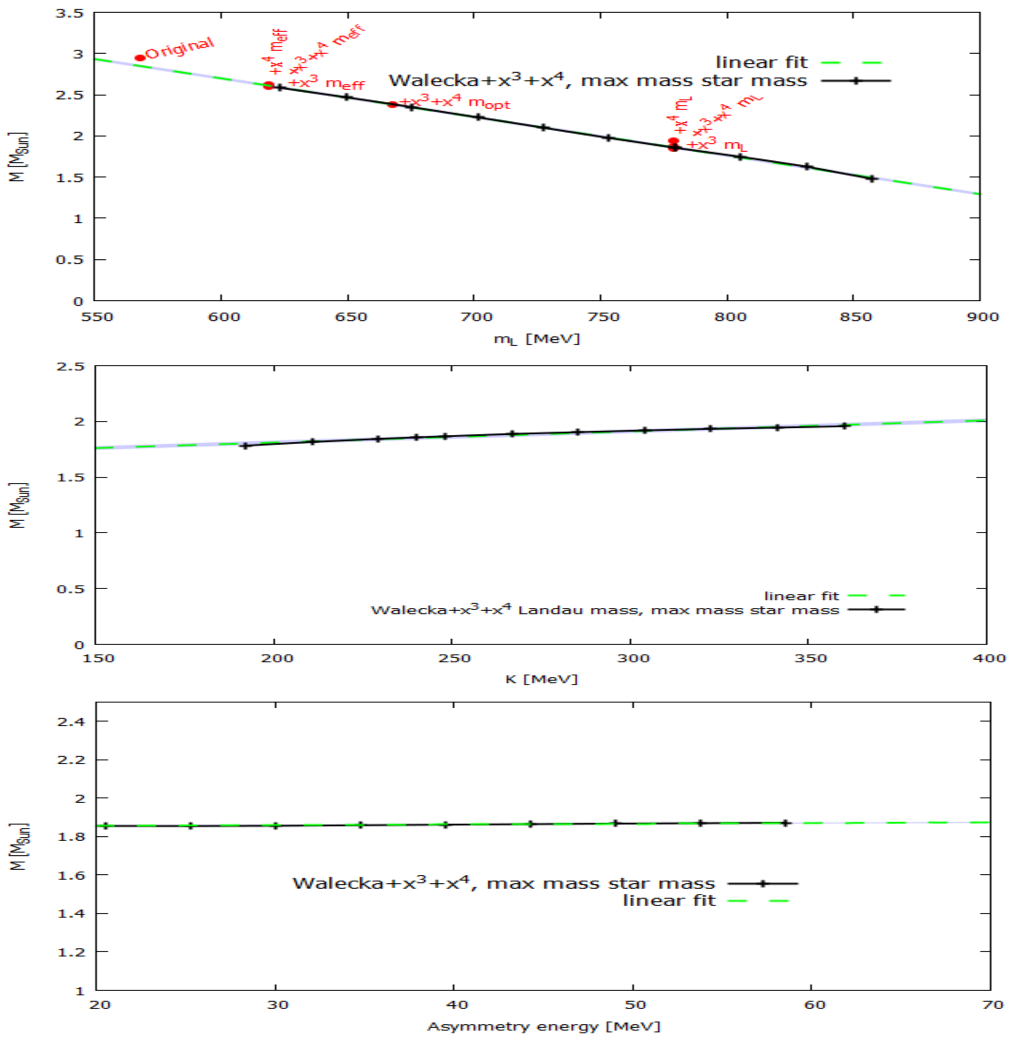


Evolution/scaling in M_{max} appears

- The M_{max} is increasing as the Landau (effective) mass is decreasing

→ Scaling by nuclear parameters

Scaling: maximum star mass vs. nuclear parameters



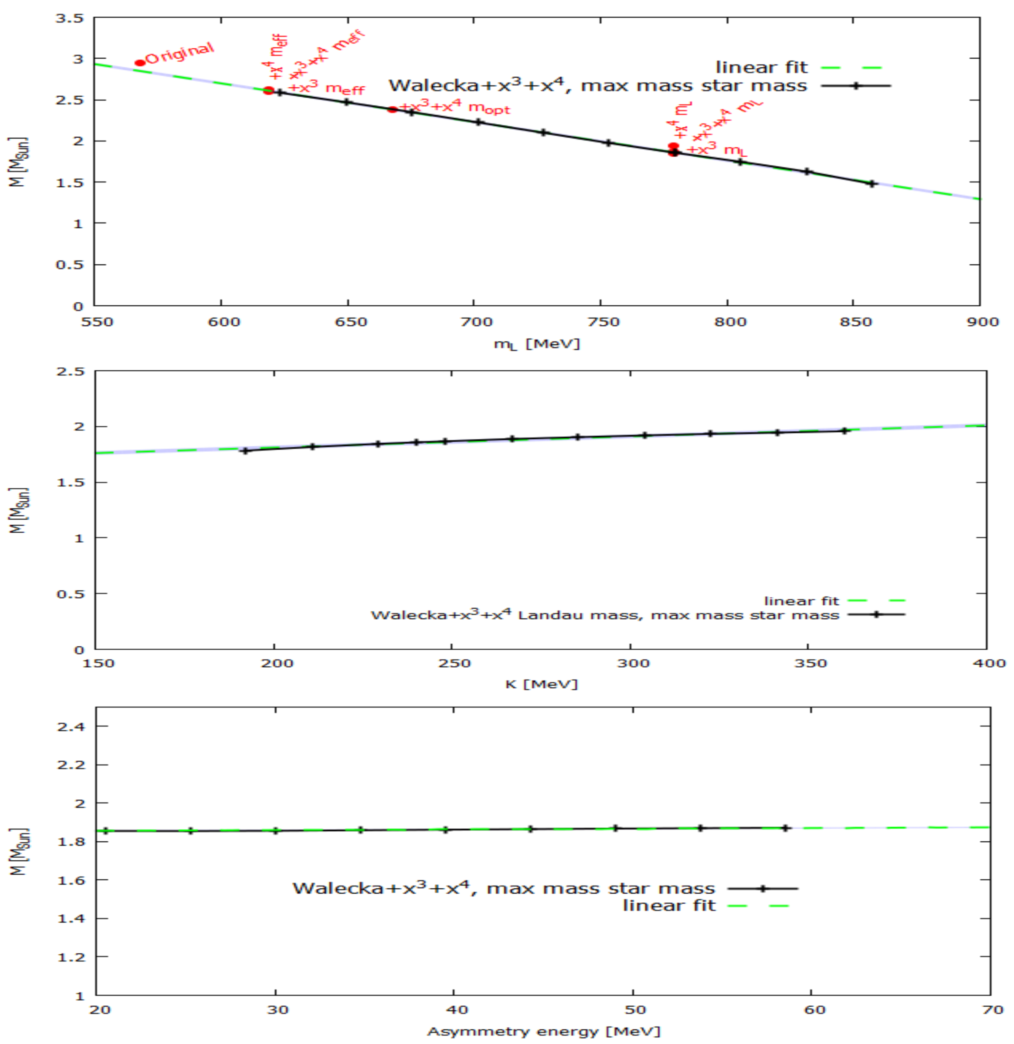
Evolution/scaling of the maximum mass/radius of the compact star

- The M_{max} is increasing as the Landau (effective) mass is decreasing

→ Scaling by nuclear parameters

- Fit errors are small < 1%
- M_{max} depends linearly by parameters
  $m_L, m_{Eff} >_{10x} K >_{10x} a_{sym}$
- Good approximation using effective mass, independently of the scalar interaction term
- Similar scaling for R_{max}

Scaling: maximum star mass vs. nuclear parameters



SYMMETRIC nuclear matter

Maximal mass (in $M_{\odot}$)

$M_{maxM} = 5.51 - 0.005\, m_L$

$M_{maxM} = 1.79 + 0.001\, K$

ASYMMETRIC nuclear matter

Maximal mass (in $M_{\odot}$)

$M_{maxM} = 5.50 - 3.64\, m_L$

$M_{maxM} = 1.61 + 0.24\, K$

$M_{maxM} = 1.85 + 0.01\, a_{sym}$

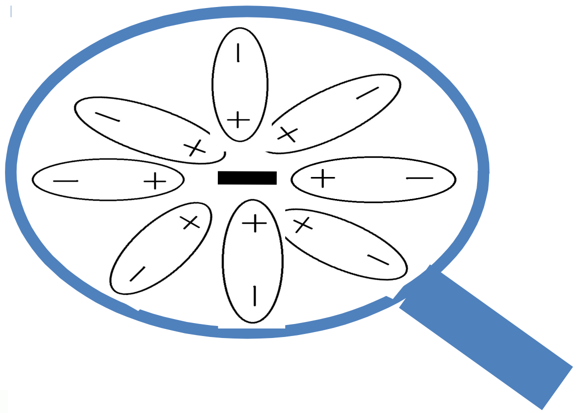
To take away...

- **Theoretical (maximal) uncertainties were tested in FRG**
  - Microscopical level (EoS, phases, compressibility): 10-25%
  - Macroscopical astrophysical level (M,R,compactness): 5-10%

- **Uncertainties by the realistic nuclear matter parameters**
  - Linear dependence on the m_L, m_{Eff} $>_{10x}$ K $>_{10x}$ a_{sym}
  - Varying m_L, m_{Eff} cause ~10% uncertainty on M and R
  - Differences on symmetric/asymmetric matter is ~1-3%

BACKUP

Motivation for FRG



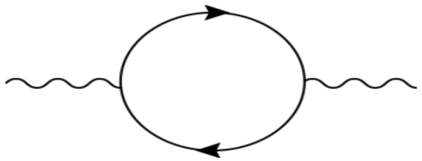
- **Observation:** Considering a point charge, which polarizes the medium seems like point charge with a modified charge.
- **Basic idea:** Due to the interaction, the measurable (effective) properties differs from the bare quantities.
- **Quantum corrections:**
  - Heisenberg uncertainty

    high-energy reaction for a short time is allowed

$$\Delta E \, \Delta t \geq \frac{\hbar}{2}$$

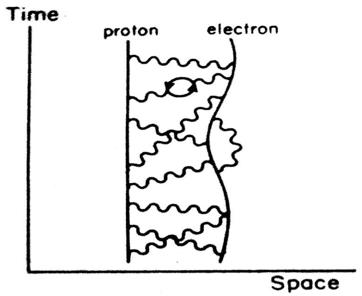
  - Pair production & annihilation

    bosonic propagator is modified due to the pair production



  - Self-interaction

    Interaction is a sum of many tiny- and self interaction



Functional Renormalization Group (FRG)

- FRG is a general non-perturbative method to determine the effective action of a system.
- **Scale dependent effective action (k scale parameter)**

$$\partial_k \Gamma_k = \frac{1}{2} \int dp^D \; STr \left[\frac{\partial_k R_k}{\Gamma_k^{(2)} + R_k} \right]$$

Wetterich equation

- **Ansatz** for the integration,
  - not need to be perturbative
  - scale-dependent coupling

$$\Gamma_k = \sum_{l=1}^{l=N} \frac{g_l(k)}{l!} \hat{O}_l$$

Functional Renormalization Group (FRG)

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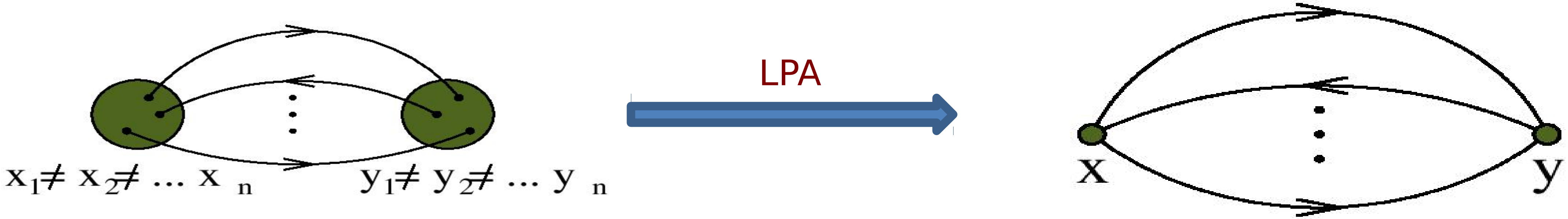
Wetterich equation

- **Regulator**
  - Determines the modes present on scale, k
  - Physics is regulator independent

Local Potential Approximation (LPA)

What does the ansatz exactly mean?

LPA is based on the assumption that the contribution of these two diagrams are close. *(momentum dependence of the vertices is suppressed)*



This implies the following ansatz for the effective action:

$$\Gamma_k[\psi] = \int d^4x \left[\frac{1}{2}\psi_i K_{k,ij}\psi_j + U_k(\psi)\right]$$

Interacting Fermi-gas at finite temperature

Ansatz for the effective action:

$$\Gamma_k[\varphi,\psi] = \int d^4x \left[\bar{\psi}\left(i\partial\!\!\!/ - g\varphi\right)\psi + \frac{1}{2}\left(\partial_\mu\varphi\right)^2 - U_k(\varphi)\right]$$

Wetterich -equation

$$\partial_k U_k = \frac{k^4}{12\pi^2}\left[\frac{1+2n_B(\omega_B)}{\omega_B} + 4\,\frac{-1+n_F(\omega_F-\mu)+n_F(\omega_F+\mu)}{\omega_F}\right]$$

Bosonic part

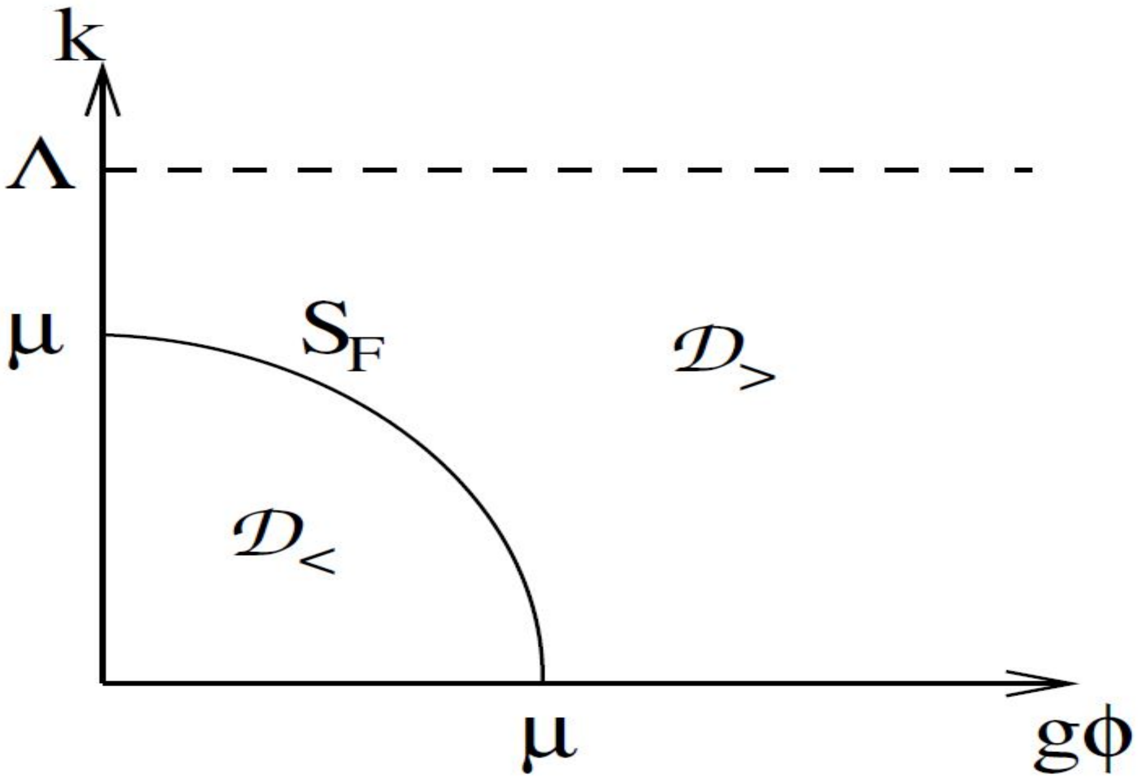
Fermionic part

$$U_\Lambda(\varphi) = \frac{m_0^2}{2}\varphi^2 + \frac{\lambda_0}{24}\varphi^4 \qquad \omega_F^2 = k^2 + g^2\varphi^2 \qquad \omega_B^2 = k^2 + \partial_\varphi^2 U \qquad n_{B/F}(\omega) = \frac{1}{1 \mp e^{-\beta\omega}}$$

Interacting Fermi-gas at zero temperature

T=0 , μ≠0 ⟹ $n_F(\omega) \to \Theta(-\omega)$

We have two equations for the two values of the step function each valid on different domain



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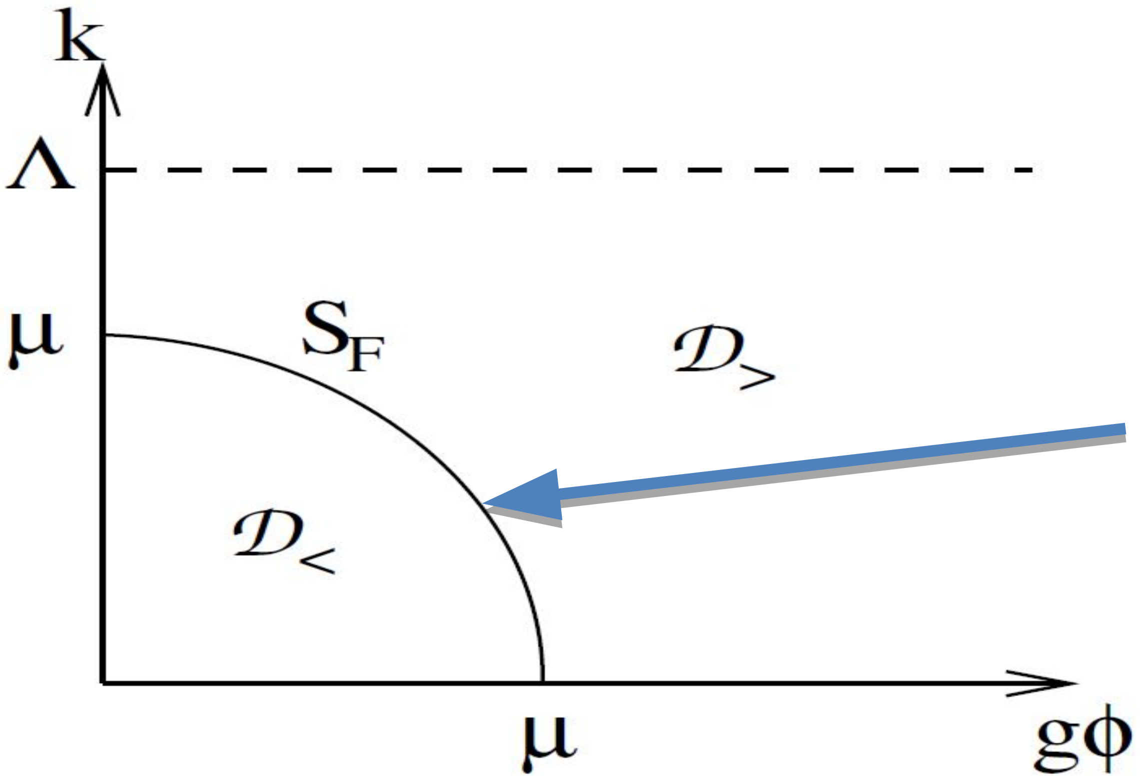


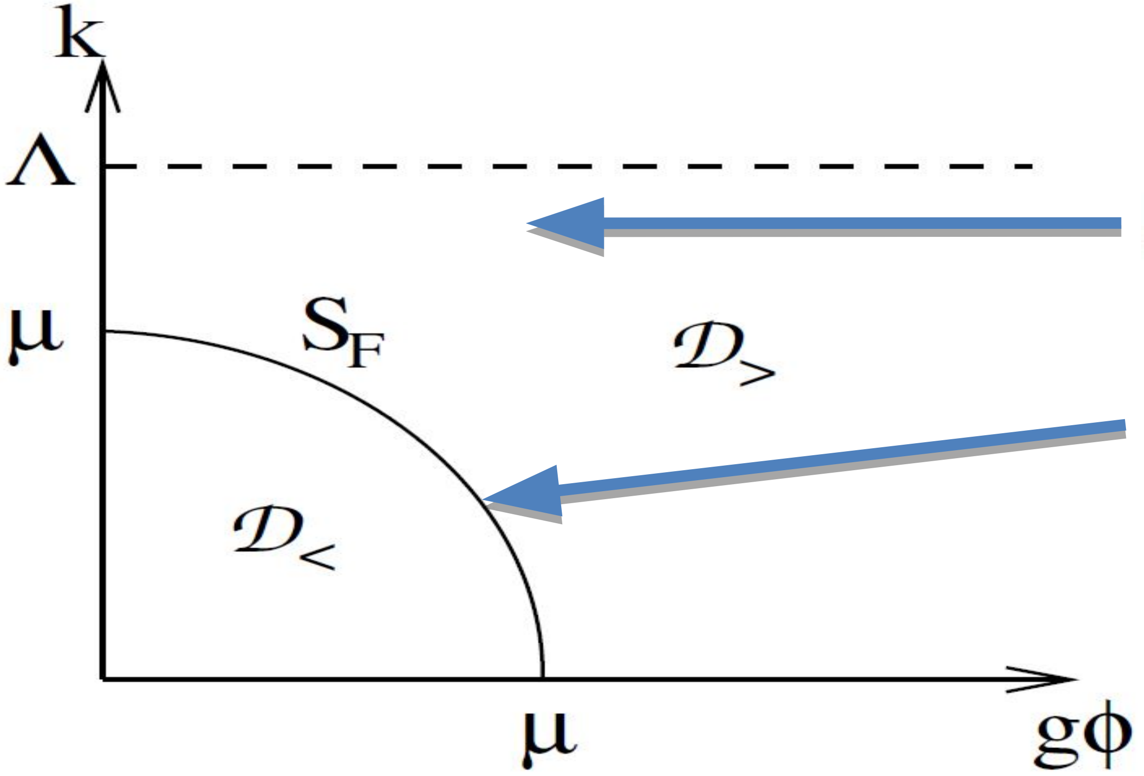
$$k_F = \sqrt{\mu^2 - g^2\varphi^2};$$

Fermi-surface

Interacting Fermi-gas at zero temperature

T=0 , μ≠0 ⟹ $n_F(\omega) \to \Theta(-\omega)$

We have two equations for the two values of the step function each valid on different domain



$$\partial_k U_k = \frac{k^4}{12\pi^2}\left[\frac{1}{\omega_B} - \frac{4}{\omega_F}\right]$$

$$k_F = \sqrt{\mu^2 - g^2\varphi^2};$$

Fermi-surface

Interacting Fermi-gas at zero temperature

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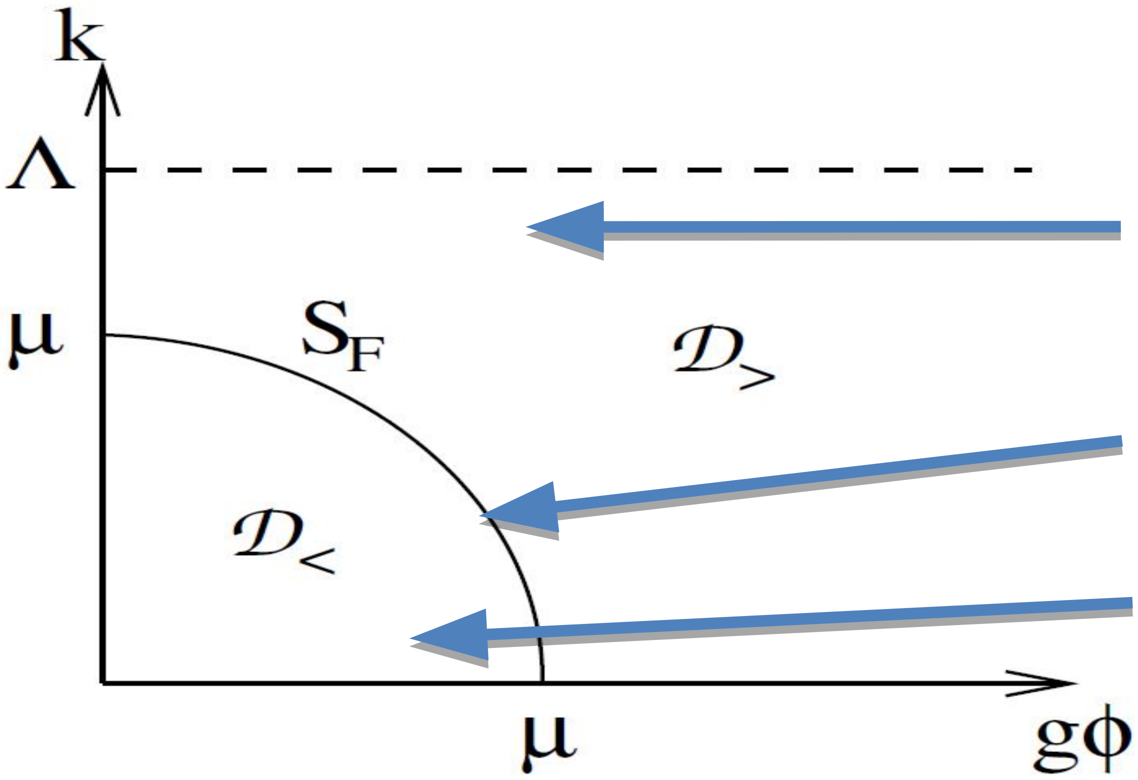


$$\partial_k U_k = \frac{k^4}{12\pi^2}\left[\frac{1}{\omega_B} - \frac{4}{\omega_F}\right]$$

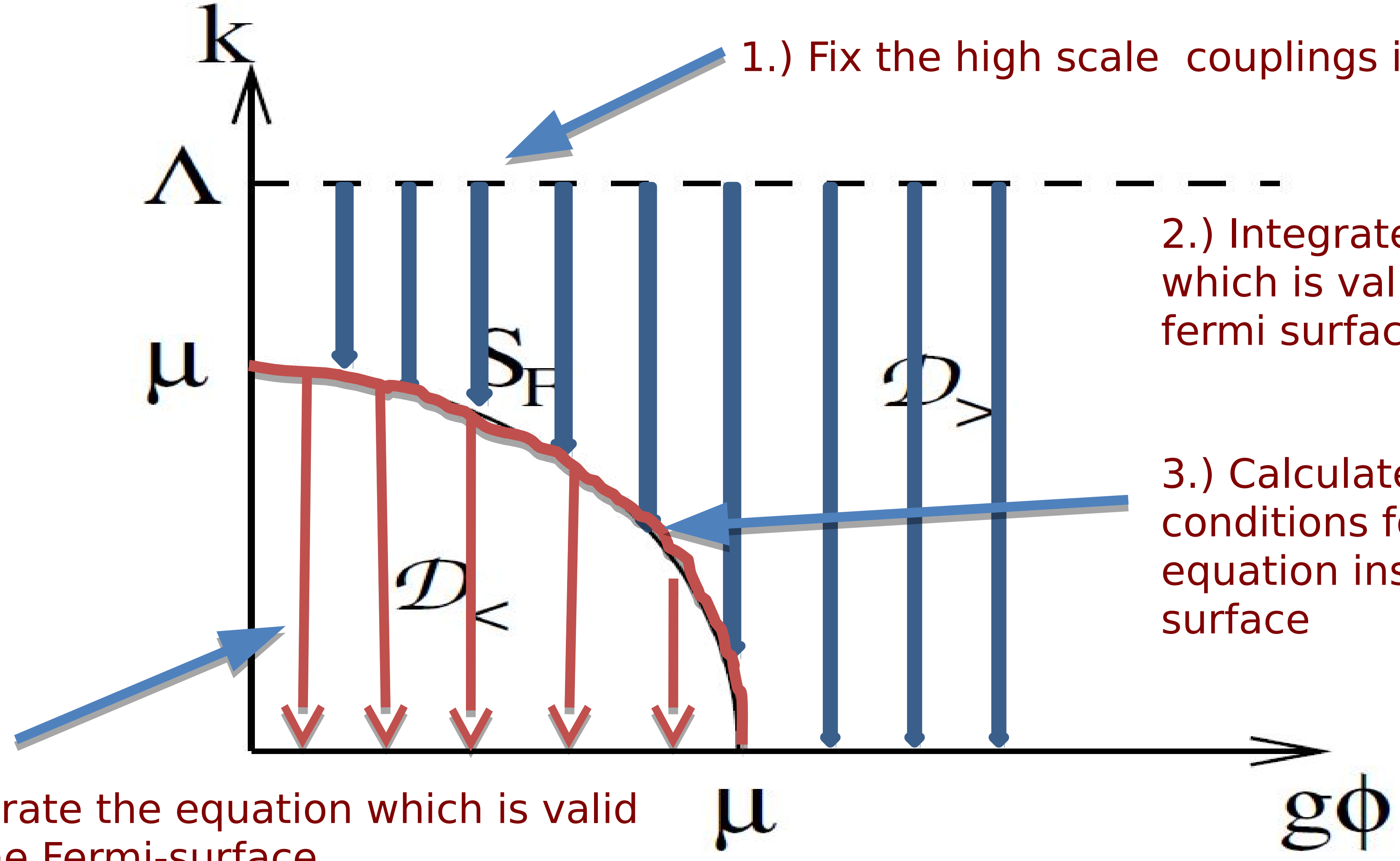
$$k_F = \sqrt{\mu^2 - g^2\varphi^2};$$

Fermi-surface

$$\partial_k U_k = \frac{k^4}{12\pi^2}\frac{1}{\omega_B}$$

Fermionic vacuum fluctuations and thermodynamic fluctuations cancel

Integration of the Wetterich-equaiton



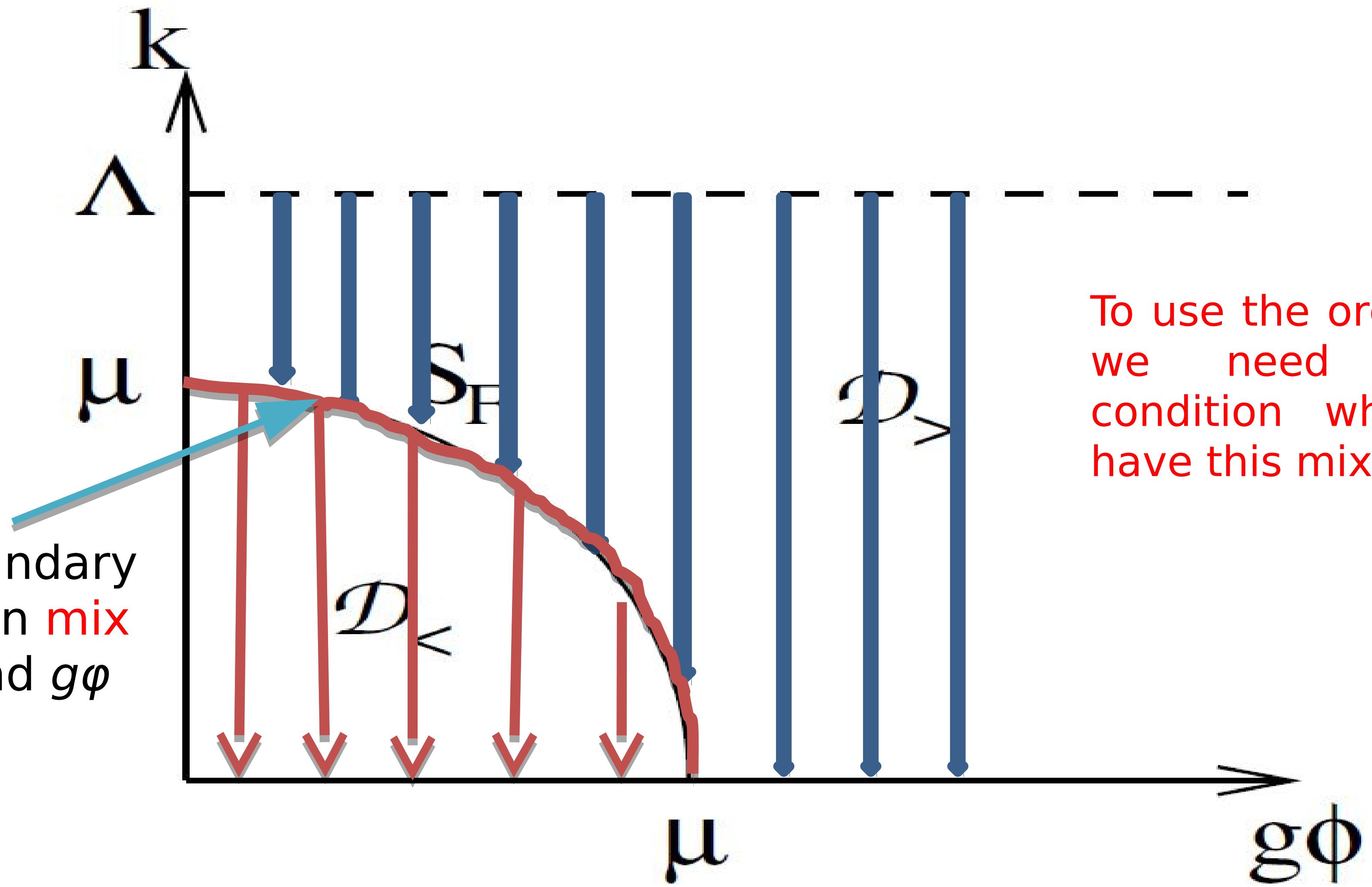
1.) Fix the high scale couplings in the theory

2.) Integrate the equation which is valid outside of the fermi surface

3.) Calculate the initial conditions for the other equation inside the fermi surface

4.) Integrate the equation which is valid below the Fermi-surface

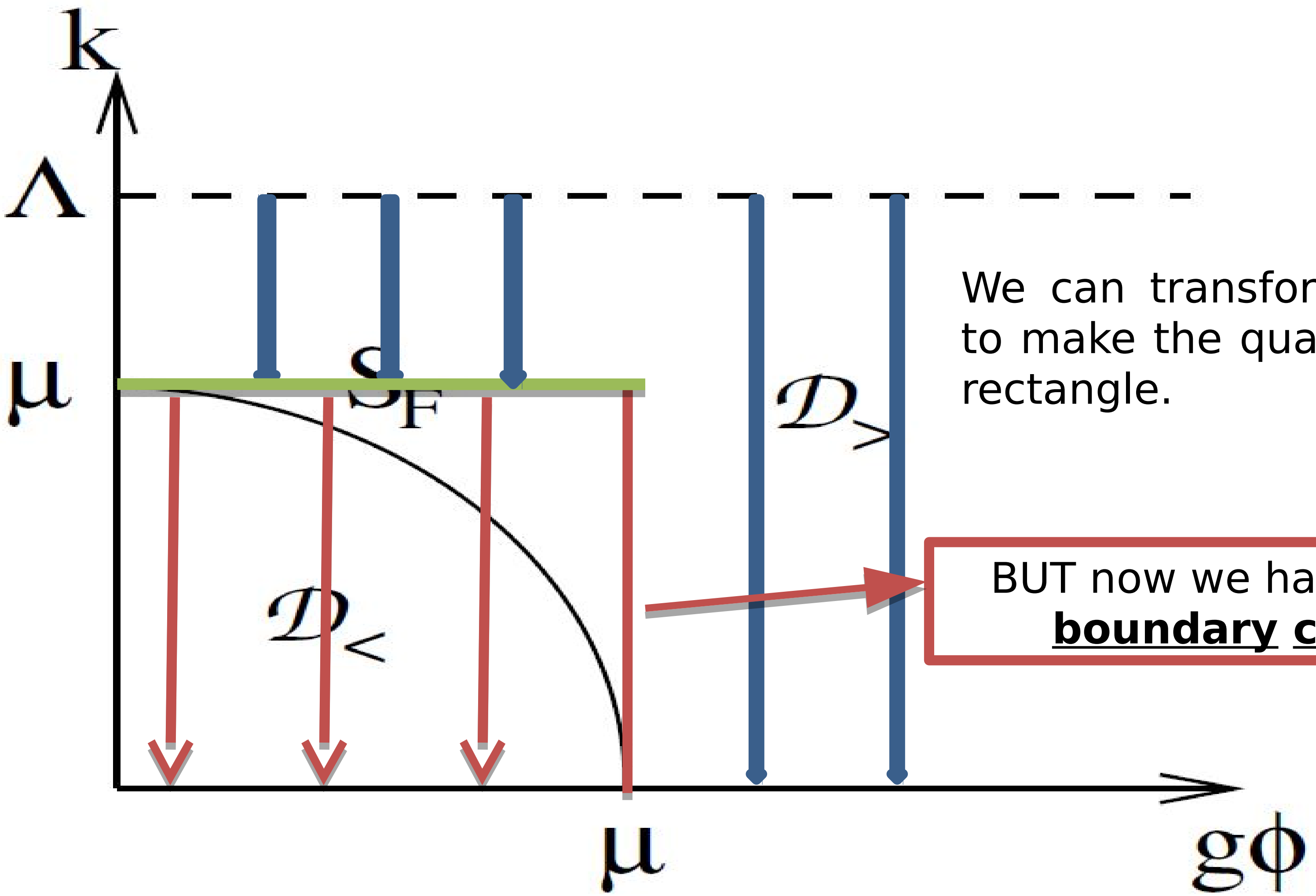
BUT…



The boundary condition mix the *k* and *gφ*

To use the orginal method we need an initial condition which do not have this mixing

Solution: Need to transform the variables

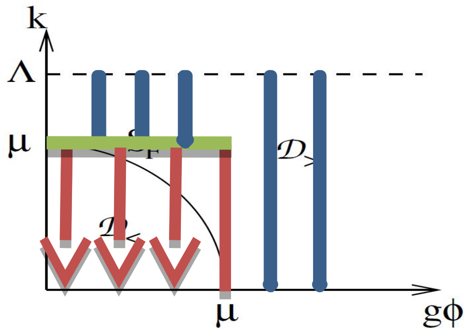


We can transform the variables to make the quarter circle into a rectangle.

BUT now we have a well defined **boundary condition** too!

Solution: Circle → Rectangle transformation

- Coordinate transformation is required with: $(k, \varphi) \mapsto (x, y)$
  - mapping the Fermi-surface to rectangle
  - Keep the symmetries of the diff. eq.
  - Circle-rectangle transformation: $x = \varphi_F(k), \quad y = \dfrac{\varphi}{x}$



- Transformation of the potential: $\tilde{U}(x, y) = V_0(x) + \tilde{u}(x, y)$

with boundary condition at the Fermi-surface, V_0

- Transformed Wetterich-eq: $x\partial_x \tilde{u} = -xV_0' + y\partial_y \tilde{u} - \dfrac{g^2(kx)^3}{12\pi^2} \dfrac{1}{\sqrt{(kx)^2 + \partial_y^2 \tilde{u}}}.$

- and the new boundary conditions: $\tilde{u}(x = 0, y) = \tilde{u}(x, y = \pm 1) = 0.$

Solution of transformed Wetterich by an orthogonal system

- Solution is expanded in an orthogonal basis to accommodate the strict boundary condition in the transformed area

$$\tilde{u}(x,y) = \sum_{n=0}^{\infty} c_n(x) h_n(y) \quad h_n(1) = 0 \quad \int_0^1 dy\, h_n(y) h_m(y) = \delta_{nm}$$

- The square root in the Wetterich-equation is also expanded:

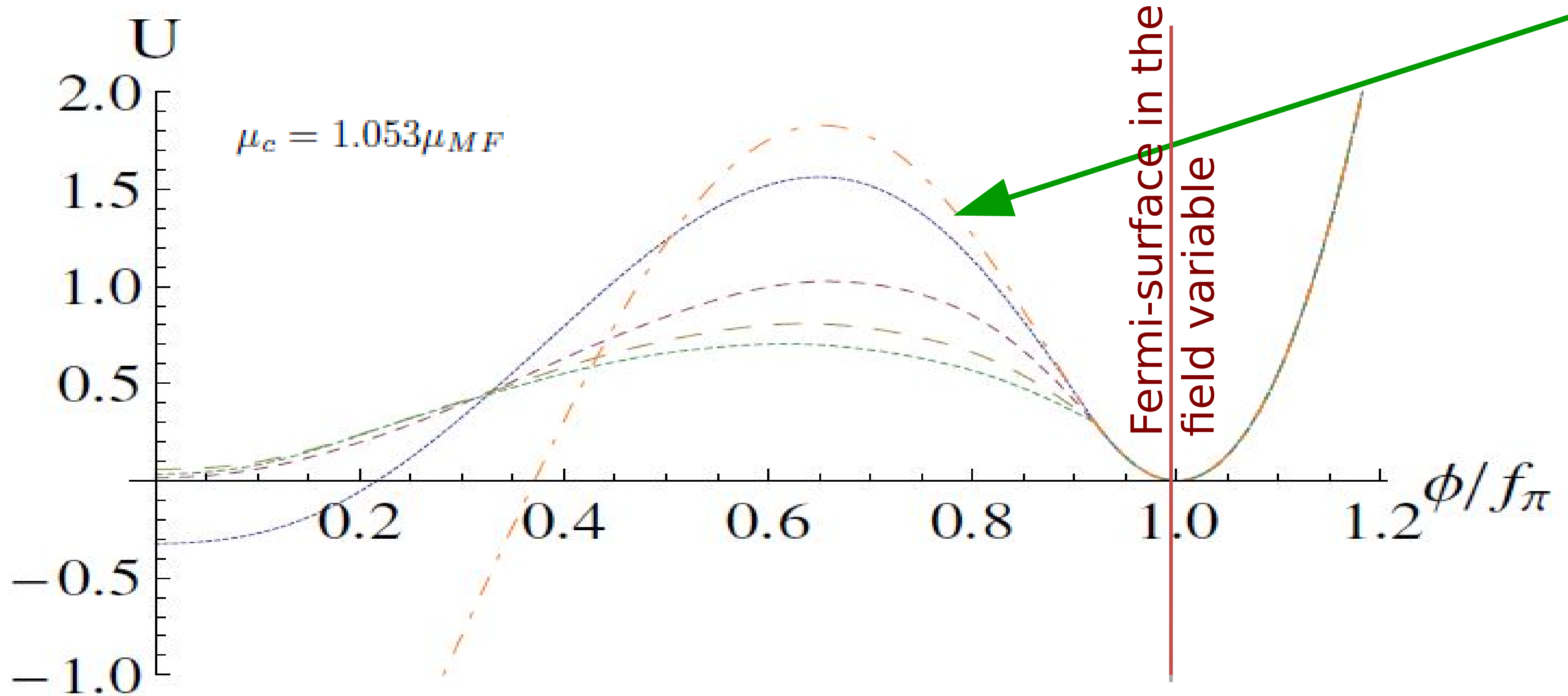
$$x c_n'(x) = \int_0^1 dy\, h_n(y) \left[-x V_0' + y \partial_y \tilde{u} - \frac{g^2 (kx)^3}{12\pi^2} \sum_{p=0}^{\infty} \binom{-1/2}{p} \frac{(\partial_y^2 \tilde{u} - M^2)^p}{\omega^{2p+1}} \right]$$

Where: $\omega^2 = (kx)^2 + M^2$

Expanded square root

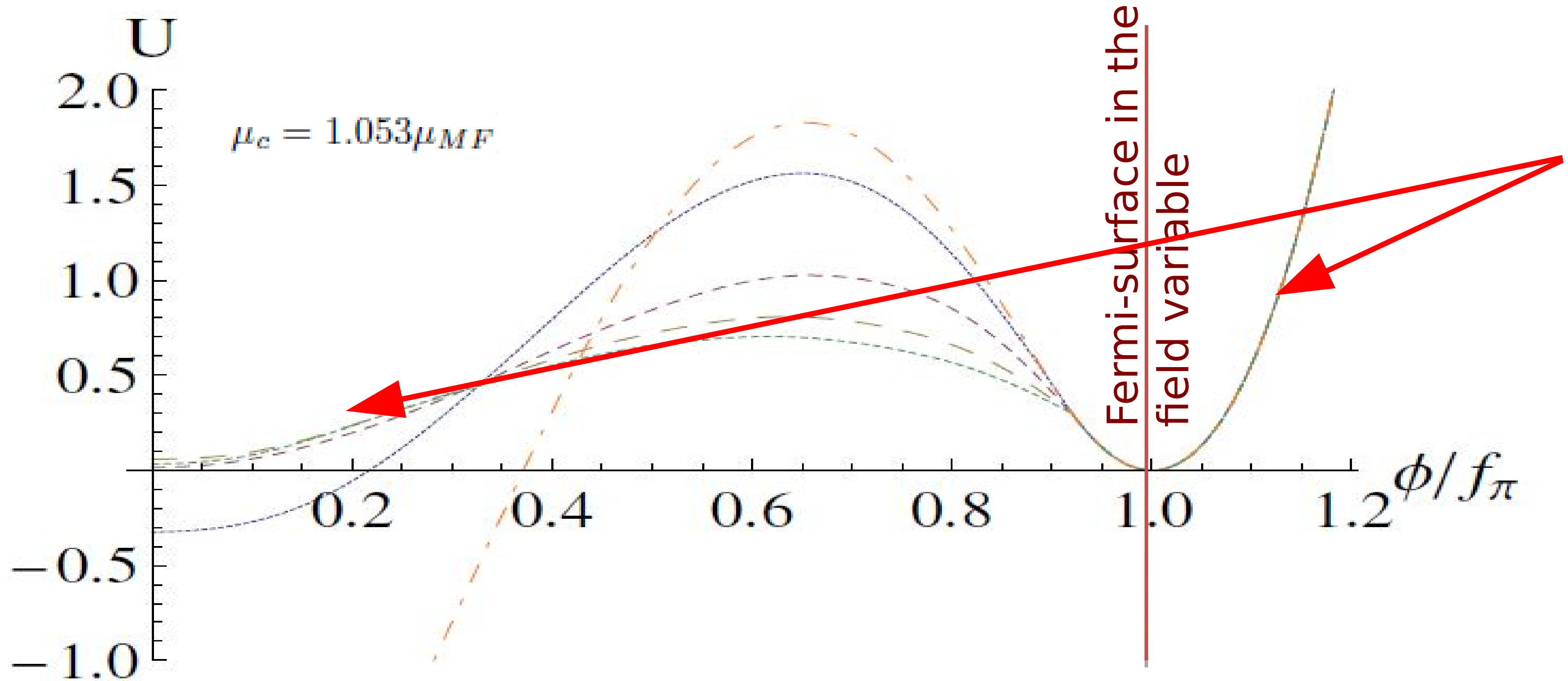
We use harmonic base: $h_n(y) = \sqrt{2} \cos q_n y, \quad q_n = (2n+1)\frac{\pi}{2}$

Result: The Effective Potential & Comparison



Potential in one-loop approximation

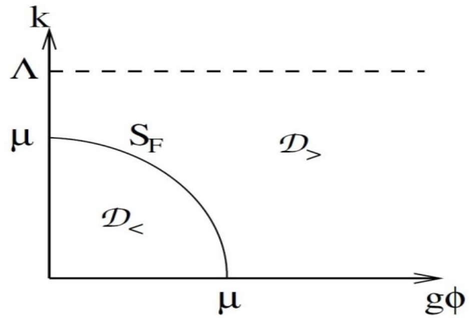
Result: The Effective Potential & Comparison



Potential in one-loop approximation

Higher orders of the Taylor-expansion for the square root converge fast where the potential is convex
→ **coarse grained action**

Solution changes only below Fermi-surface, since switch to another equation



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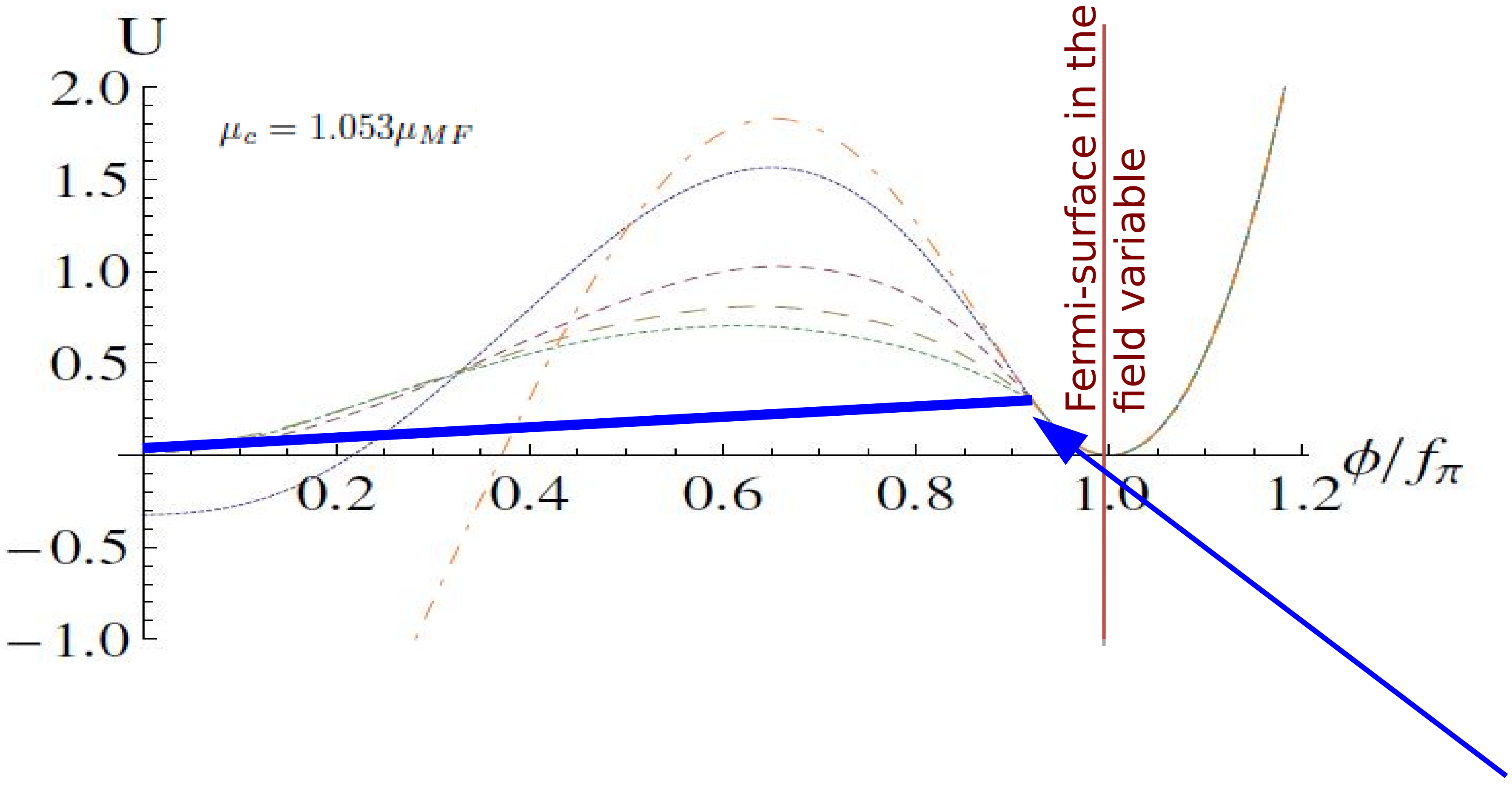


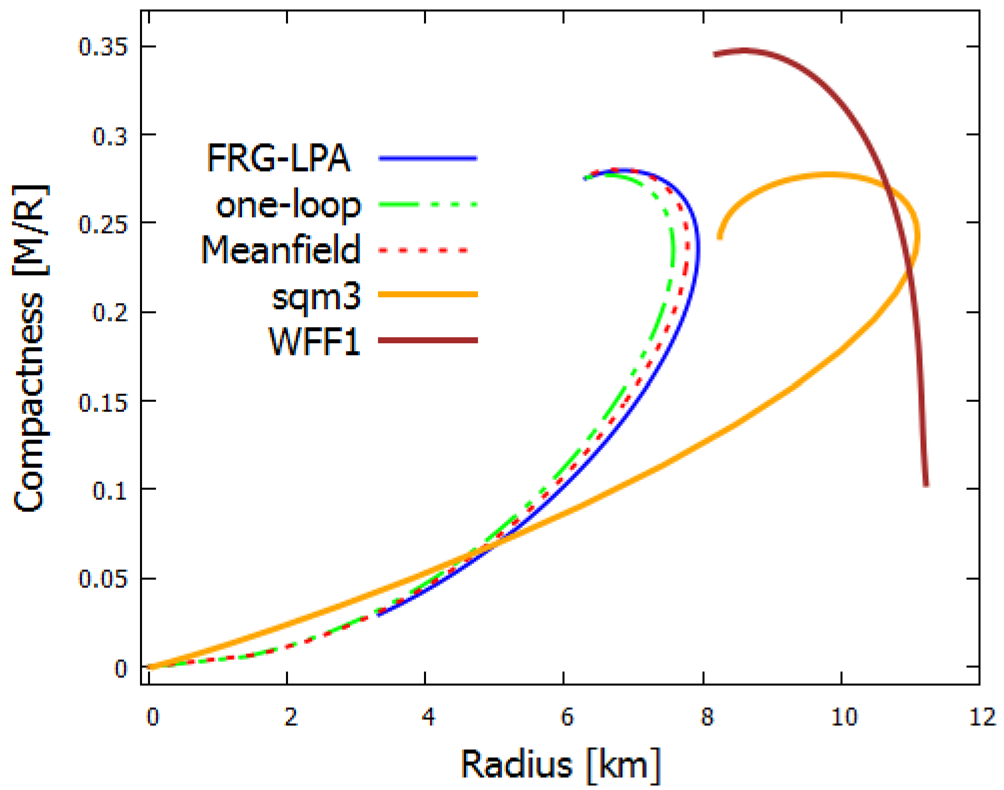
Potential in one-loop approximation

Higher orders of the Taylor-expansion for the square root converge fast where the potential is **convex**
→ **coarse grained action**

In the **concave** part of the potential solution is slowly converges to a straight line, because the free energy (effective potential) must be convex from thermodynamical reasons
→ **Maxwell construction**

Test: Can we test this by observations?

- Compare Compactness by FRG, MF, 1L, SQM3, and WFF1 EoS



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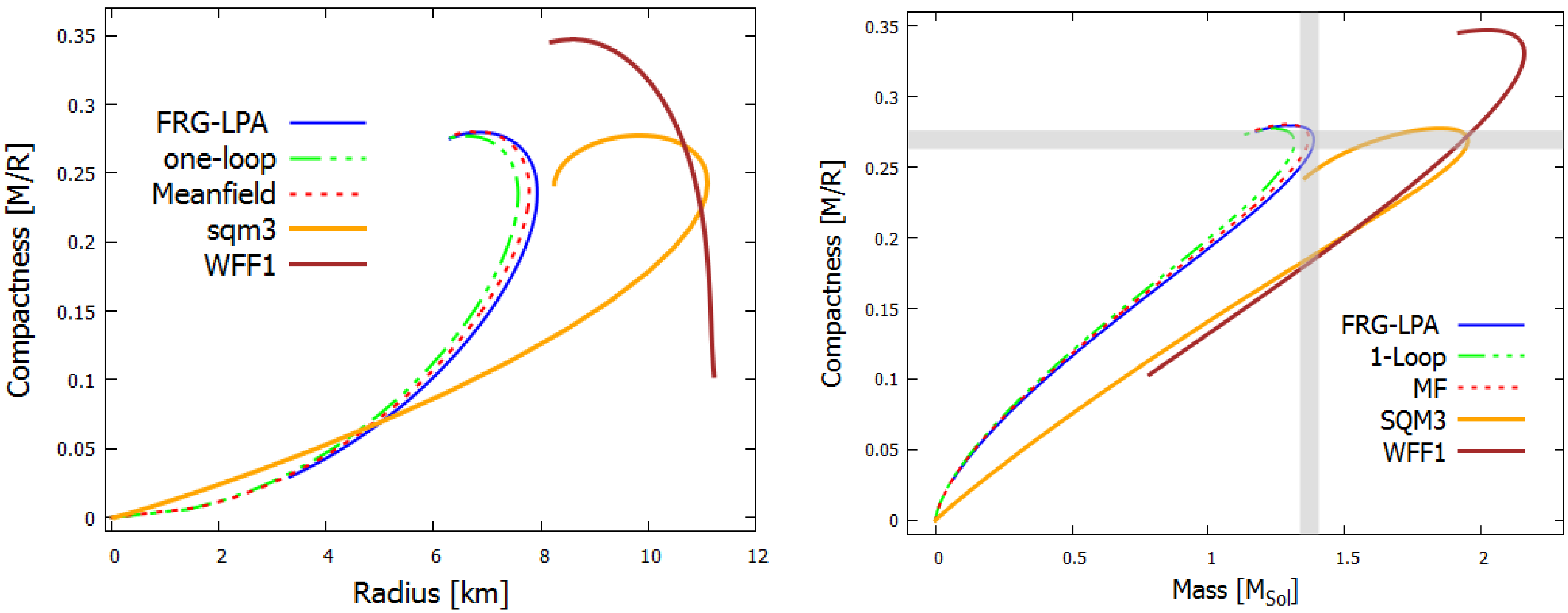

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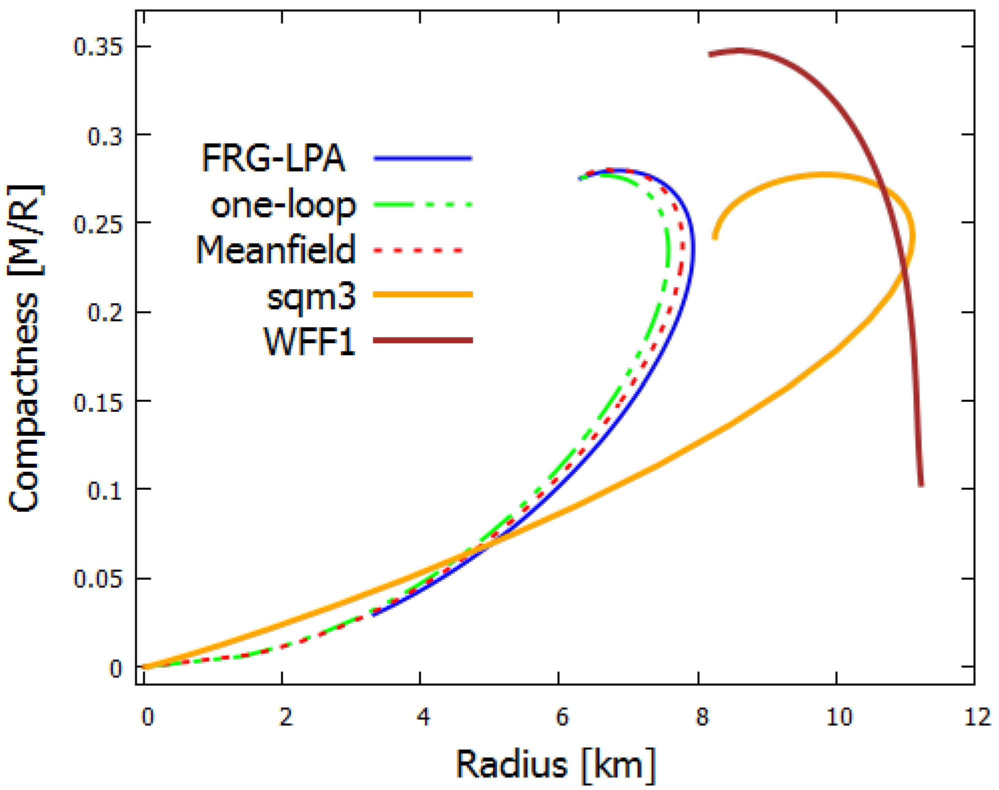

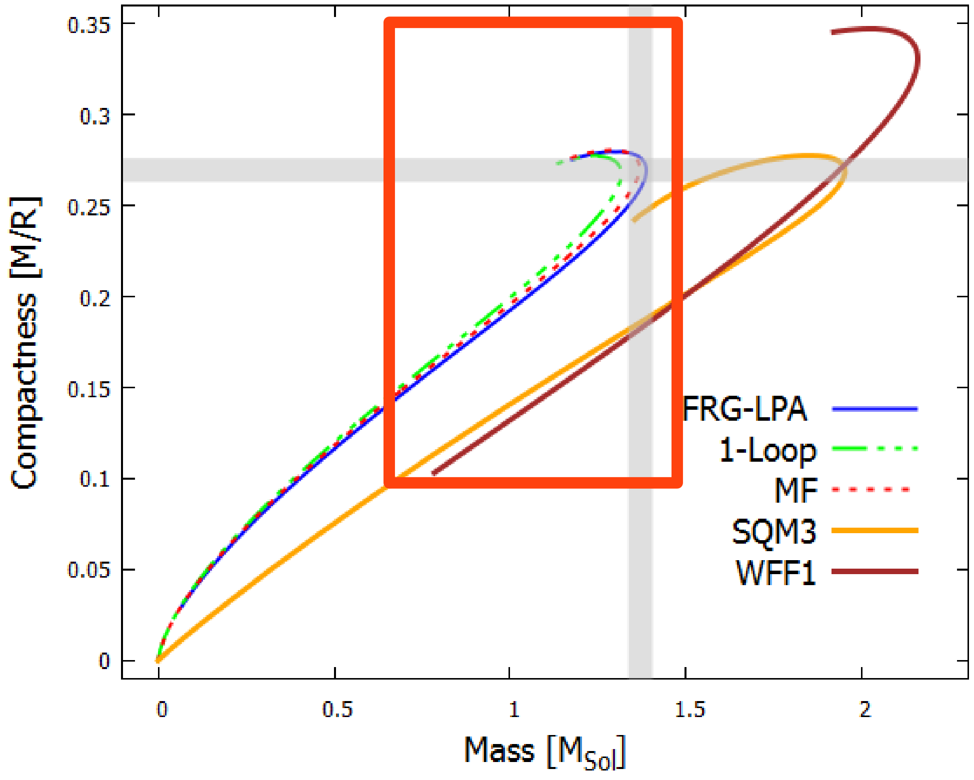

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