Tsallis-like Fragmentation Functions for High Energy Collisions

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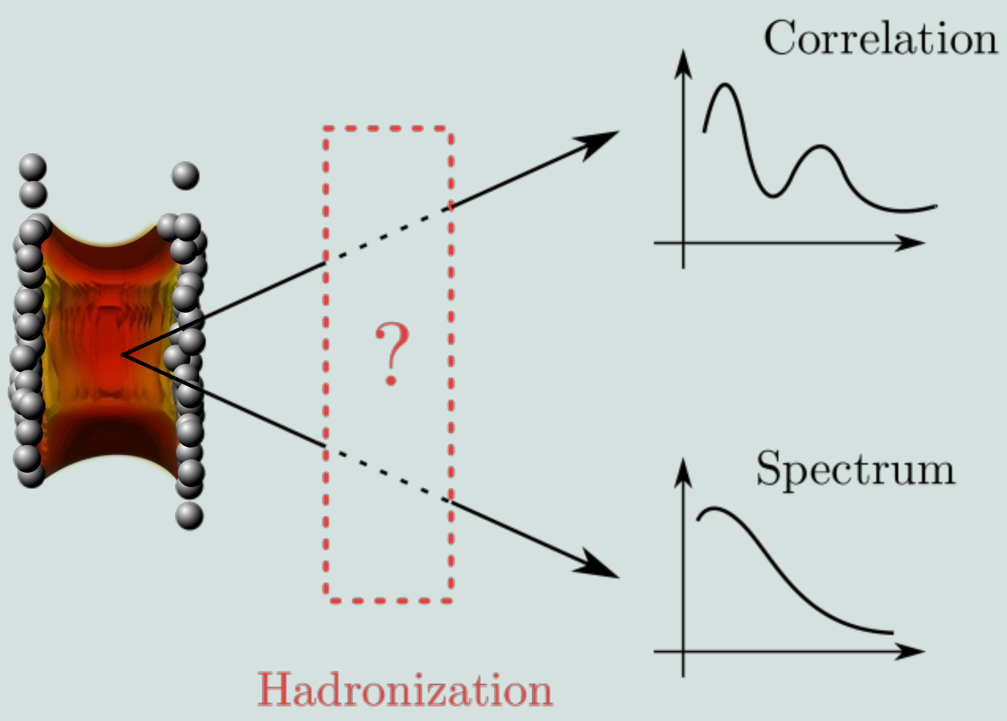
Motivation

To predict measurable quantities in any model, hadrons are needed. The process, when quarks and gluon confine into hadrons is the **hadronization**, which is unresolved problem.

Methods to describe the formation of hadronic final states in high-energy collisions.

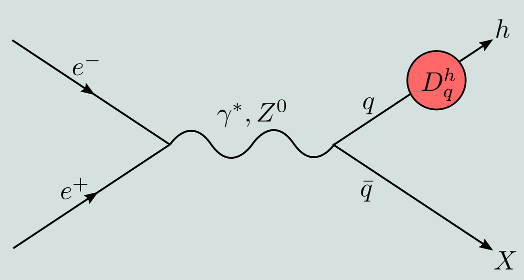
- Freeze-out, like the Cooper–Frye formula in hydrodynamics
- Dynamical fragmentation, like the Lund Model in MC generators
- Recombination or quark coalescence models
- **Statistical fragmentation**, such as the Feynman–Field Model



Fragmentation function D_i^h, describes the probability of a parton i forms a hadron h. This non-perturbative process is formulated phenomenological and parametrized in e^-e^+ annihilation.

At leading order (LO), $e^- + e^+ \to h + X$ spectra is calculated by

$$\frac{\mathrm{d}\sigma(e^-e^+ \to hX)}{\mathrm{d}z} = \sum_i \sigma_0^i(s) D_i^h(z, Q^2), \qquad (1)$$



where σ_0 is the partonic $2 \to 2$ cross section, energy scale is $Q = \sqrt{s}/2$ and the energy fraction is $z = E_h/Q$.

The Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) equation provides the scale evolution of the fragmentation functions at any Q values

$$\frac{\mathrm{d}D_i^h(z,\mu)}{\mathrm{d}\log Q^2} = \sum_j \int_z^1 \frac{\mathrm{d}x}{x} P_{ij}\left(z/x, Q^2\right) D_i^h\left(x, Q^2\right), \qquad (2)$$

where P_{ij} are the splitting functions.

Hadronization by Statistical Fragmentation

Fragmentation function is a parametrized phenomenological function, providing the probability of a parton i confine into a hadron h. Its form is motivated by the measured spectra.

Standard parametrization

- "QCD motivated" formula (polynomial approximation) [1, 2]

$$D_i^h(z, Q) = N_i^h z^{\alpha_i^h}(1-z)^{\beta_i^h} \qquad (3)$$

- Theoretical predictions for N, α and β differs from the experiment data
- **No physical meaning of parameters** N, α and β
- Q-evolved formula is NOT polynomial-like
- Poor agreement with spectra at small momenta $z < 0.1$

New method

- Statistically motivated Tsallis-like formula from non-extensive study of jets [3, 4]

$$D_i^h(z, Q) = N_i^h(1-z)\left[1 - \frac{q_i^h - 1}{T_i^h}\log(1-z)\right]^{-\frac{1}{q_i^h - 1}} \qquad (4)$$

- **Parameters with physical meaning**:
  - $q \neq 1$ domination of correlations and fluctuations inside the hadronizing system
  - T temperature of the highly correlated hadronizing system
- Theoretical predictions: $q_{jet} = 1 - 2$, $T_{jet} = 1 - 10^3$ MeV from non-extensive jet studies, depending on the energy and system size
- Q-evolved formula is NOT Tsallis-like
- Looking for better results at low z with the same number of parameters

References

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Results

Tsallis-like fragmentation parametrization

A new parametrization for fragmentation function in LO is made by fitting $e^- + e^+ \to \pi + X$ spectra from Ref. [5-8].

Properties of the parametrization (Fig. 3):

- statistically motivated and describes data
- better at small momenta (better χ^2)
- needs no further parameter for **better fit!**
- agrees with predictions: $q_{fit} = 1.1 - 1.5$ and $T_{fit} = 10 - 100$ MeV **parameters are physical!**
- small differences at low energy (Fig. 4).

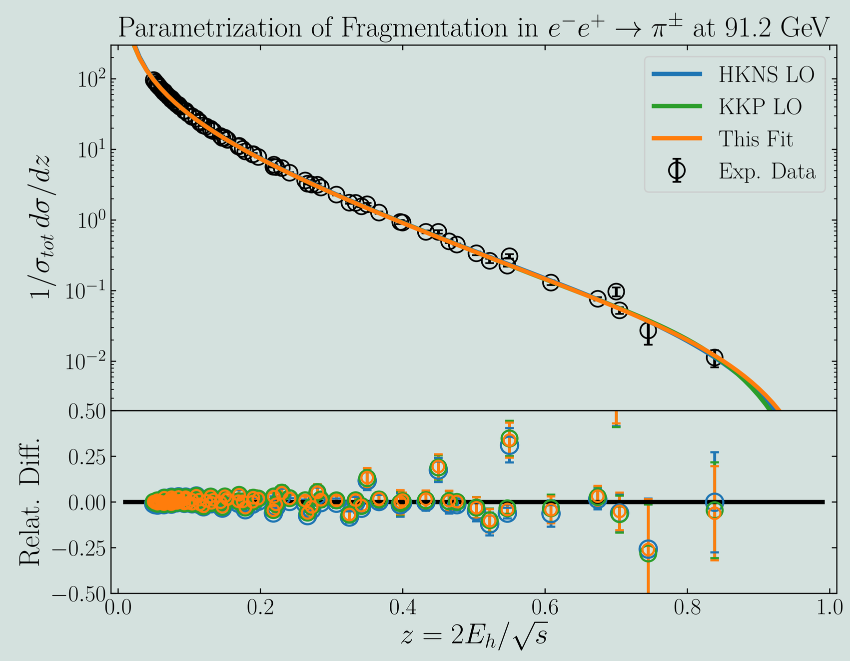

Fig. 3: Pion spectra and calculations with our standard fragmentation functions [1, 2].

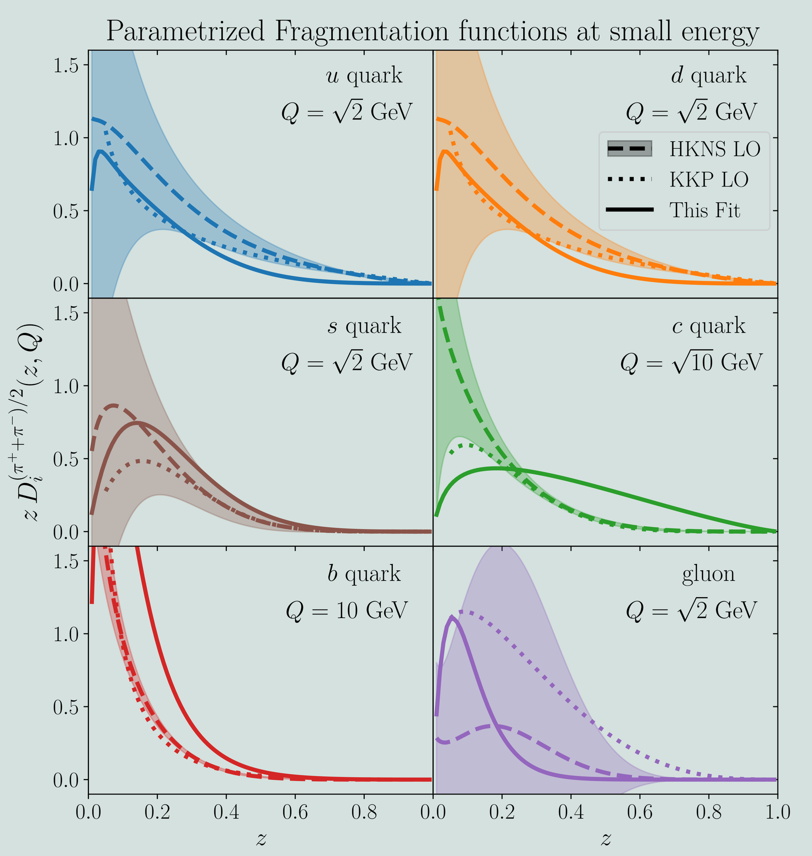

Fig. 4: Our fragmentation compared to standard ones at small Q. Good agreement between models!

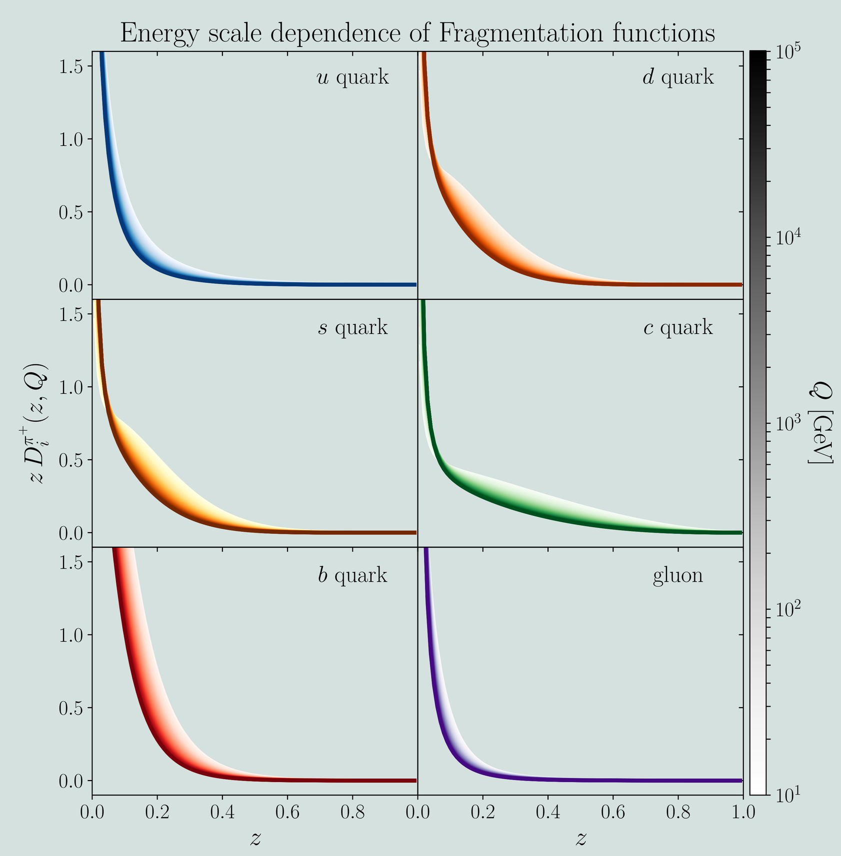

Fig. 5: Q evolution of our fragmentation by solving DGLAP. Initial differences are vanishing at high Q.

Cross check: sum rule (Fig. 6):

- Probability of a parton confinement

$$P_i^h(Q) = \int_0^1 \mathrm{d}z\, z D_i^h(z, Q)$$

- Total sum $\sum_h P_i^h \equiv 1$ (any parton confine into a hadron)
- Larger contribution for constituent and light quarks
- Weak energy dependence is similar to [1]
- Difference between fragmentation models vanishes at high energy (DGLAP clears initial differences) see Fig.5

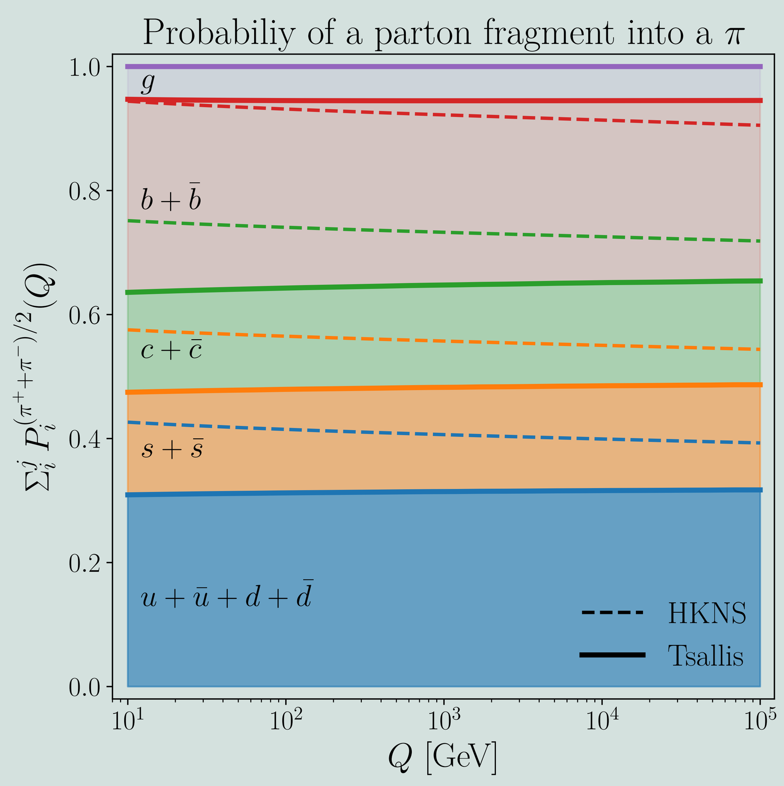

Fig. 6: The fraction of partonic channels hadronize into a pion at scale, Q compared to standard one [1]

Conclusion

- Non-extensive, Tsallis–Pareto-like fragmentation function parametrization is developed
- Physically more motivated form and parameters than standard ones
- Better fits and good performance at small z values
- Parametrization and grids are available at `https://www.kfki.hu/~takacsa/`
- Student is looking Phd. position, see more at the webpage above

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See more at arXiv: 1805.

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