Correspondence of Many-flavor Limit and Kaluza-Klein Degrees of Freedom in the Description of Compact Stars

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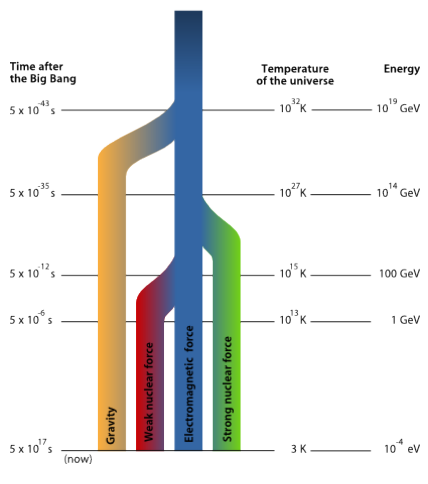
Abstract

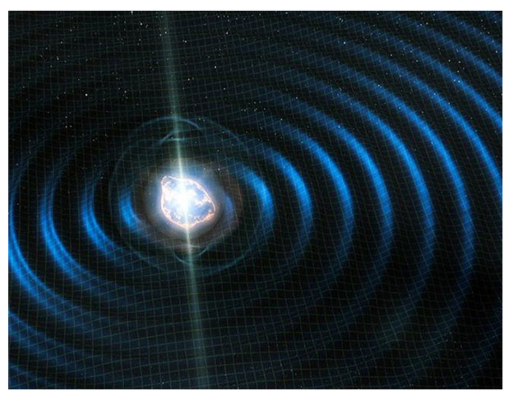
However, there is no direct observation of the inner structure of a compact star, some physical properties like the measured mass-radius relation, moment of inertia, rotation period, magnetic field and the soon-to-be-available gravitational wave observations support to build and parametrize realistic equations of state (EoS) in the non-perturbative and high-density QCD (Quantum Chromodynamics) regime. We present the correspondence between (non)-interacting multi-hadron fermion star equation of state in the many-flavor limit and the degrees of freedom of a Kaluza–Klein compact star. Many flavors can be interpreted in this framework as one extra compactified spatial dimension with various, more-and-more massive hadron state excitations. The effect of increasing the degrees of freedom was investigated on the equation of state and in connection with the mass-radius relation, M(R). This theory led us to investigate gravitational theories beyond the post-Newtonian case. Our aim is to present whether we would be able to measure the effect of these extra dimensions by gravitational wave detectors in the future.

Motivation: extra dimensions in compact stars

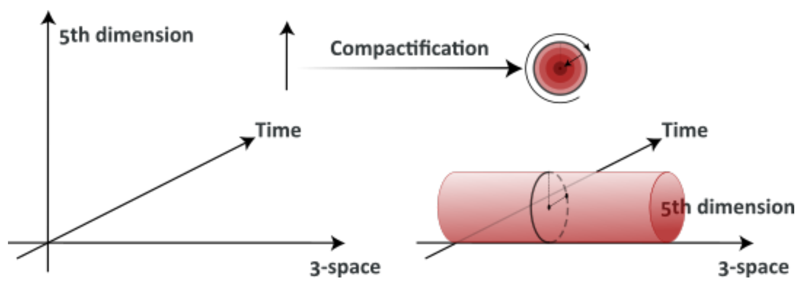
- Geometrized theoretical descriptions of the fundamental interactions lead us to get closer to the unification of the principle theories below the energies of the Planck scale. The idea of introducing compactified spatial dimensions originated by Th. Kaluza and O. Klein.
- E.g.: the Grand Unified Theory (GUT) at high energy merges the three interactions of the Standard Model into one framework. By introducing new dimensions the energy scale of GUT can be shifted to lower energy magnitudes ~O(TeV).
- Observed symmetries of the standard "low-energy" physical world suggest a space-time with 3 macroscopical spatial dimensions. Furthermore this can not exclude the existence of compactified extra spatial dimensions at microscopical scales at extreme energies.
- A compact star constructed in a 1+4 dimensional Kaluza–Klein spacetime must behave as a standard 1+3 dimensional object as well for a 1+3 dimensional observer.
- Recent multi-wavelength electromagnetic and gravitational observations led us to explore the deviation from the Newtonian gravitation. In the Kaluza–Klein model, comparison between non-interacting multi-hadron fermion star equation of state in the many-flavor limit and the degrees of freedom of a Kaluza–Klein compact star can be done.



This is the basis of the Grand Unified Theory (GUT) ideas.



Light and gravitational waves from a recent neutron star merger (illustrated) suggested that there are only three large dimensions of spacetime into which gravity can penetrate. Pardo et al. (2018)



The 5th dimension (which would be a 4th space-like dimension) is curled up in a circle of an extremely small radius (10^{-35} m): in every point of space time there is one extra dimension rolled-up into a circle.

Special analytical solution in 1+4 dimensional KK spacetime

- When describing the inner structure of a compact spherically static compact object in a 1+3+d_c dimensional spacetime in a perfect fluid approximation with isotropy. The following conditions are required for the consistency:
  - 1+(3+d_c) dimensional space-time: dimensions are space-like, except first one which is time-like
  - GR is the same as in 1+3D ⇨ "Equivalence Principle" is unchanged in the improved theory
  - All causality postulates are the same as in 1+3 D, including even lightcone structure
  - Extra space-like d_c dimensions are microscopical
  - There is complete Killing-symmetry in the d_c-dimensional microscopical subspace

The above consequences in the 1+3+1_c dimensional spactime yield a diagonal metric, with a 5[th] diagonal component which is independent from the other coordinates:

$$g_{\mu\nu} = \mathrm{diag}(e^{2\nu}, -e^{2\lambda}, -r^2, -r^2\sin^2\theta, e^{2\phi})$$

Due to cosmological reasons an anisotropy (p_5 pressure) must be chosen for the extra direction

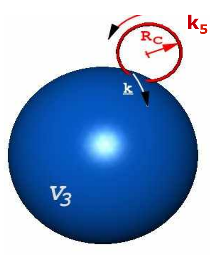
$$T_{\mu\nu} = \epsilon u_\mu u_\nu - p(g_{\mu\nu} - u_\mu u_\nu + v_\mu v_\nu) - p_5 v_\mu v_\nu \quad \text{and} \quad p_5 \neq p$$

The ϑ, λ, ν and φ are radial functions and u_μ, u_ν and v_μ, v_ν are the five-velocities set as (0,0,0,0,v_5). Using dφ/dr=0 there is an analitic solution of Einstein equations which leads to a 5D TOV (Tolman–Oppenheimer–Volkov) equation.

Equation of state in the Kaluza–Klein world

The states of the extra dimensional framework

- In the Kaluza-Klein theory the geometrical degrees of freedom appear as new excited states of the particle states.
- Particles with high enough energy are able to move into the extra microscopical spatial dimension.



- Motion into the 5[th] dimension (momentum k_5) generates an extra mass term what appears as "excited mass", in the 4-dimensional description:

$$E_5 = \sqrt{k^2 + k_5^2 + m^2} = \sqrt{k^2 + m^2}$$

- Introducing the extra microscopical dimension as a circle with a compactification radius R_C, the topology of the spacetime generates a periodical boundary condition in the 5[th] direction for k_5 momentum component by the wave function:

$$\psi(x_5) \approx e^{ik_5x_5} \quad \text{and} \quad \psi(x_5 + 2\pi R_C) \sim \psi(x_5) \quad \Rightarrow \quad k_5 = \frac{n}{R_C}$$

- The increased particle mass is connected to the radius of the compactified extra dimension R_C and the n[th] excitation level refers to the movement within the 5[th] dimension.

$$\tilde{m}^2 = \left(\frac{n}{R_C}\right)^2 + m^2$$

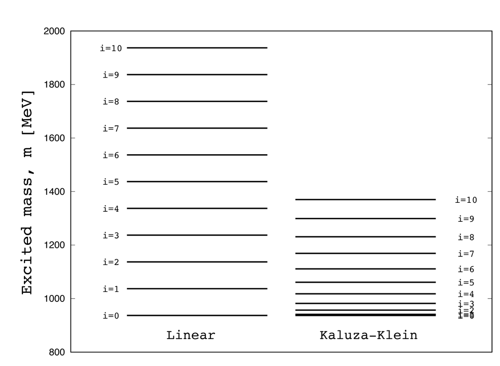
$\tilde{m}$ heavy baryon mass
m light baryon mass
n excitation number

in our simplified approach ⇨

$$m_i = m_0 + i\Delta m$$

notation change : $n \to i$ excitation number

- Degrees of freedom of the theory are fermions (baryons), but with linearly increasing excited masses, and we set the neutron mass as the base state with m_0= 936MeV and set of various Δm from the 0.1MeV-1.0GeV interval, as the excitation gap.
- Higher excitation results more massive particles (baryons), but larger R_C turns the excited state structure more denser.



The energy levels for the constant-gap linear (left) and Kaluza–Klein (right) for ground state, m_0 = 936.9 MeV, and mass gap, Δm = hc/R_C = 100 MeV with excitation number, i ∈ [1 : 10].

EoS of the non-interacting fermion matter in 5 dimension

- The equation of state (EoS) of a non-interacting fermion star with multi-hadron components is given by the thermodynamical potential:

$$\Omega = -V \sum_i \frac{g_i}{\beta} \int_0^\infty \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \times \left[\ln\left(1 + e^{-\beta(E_i - \mu_i)}\right) + \ln\left(1 + e^{-\beta(E_i + \mu_i)}\right)\right]$$

- where g_i = 2 is the spin multiplicity, $E_i = \sqrt{k^2 - m_i^2}$ is the energy of the fermion *i* with momentum *k* mass m_i in volume, *V* at inverse temperature, β. Sum is required for all degree of freedom. After the formation, stable compact stars are cold object, its typical temperature is several order of magnitude lower than the bounding energy of their nuclei, thus the zero temperature limit (T→0) is appropriate, and turns to

$$\Omega|_{T=0} = \begin{cases} -\sum_i g_i V f(m_i, \mu_i), & \text{if } m_i < \mu_i \\ 0, & \text{if } m_i \geq \mu_i \end{cases}$$

- where we have introduced the Fermi–Dirac distribution

$$f(m_i, \mu_i) := \frac{m_i^4}{24\pi^2} \times \left[\frac{\mu_i}{m_i}\sqrt{\frac{\mu_i^2}{m_i^2} - 1}\left(\frac{\mu_i^2}{m_i^2} - \frac{5}{2}\right) + \frac{3}{2}\ln\left|\frac{\mu_i}{m_i} + \sqrt{\frac{\mu_i^2}{m_i^2} - 1}\right|\right]$$

- Following the thermodynamical rules, one can calculate the pressure, p_i, the energy density, ε_i, and the particle number density, n_i
- Taking the sum for the particle components *i*, the total pressure and energy density is deduced, $p = \sum_i p_i$ and $\epsilon = \sum_i \epsilon_i$ respectively. Taking the full volume, $N = V\sum_i n_i$, the total number of particles can be also obtained.

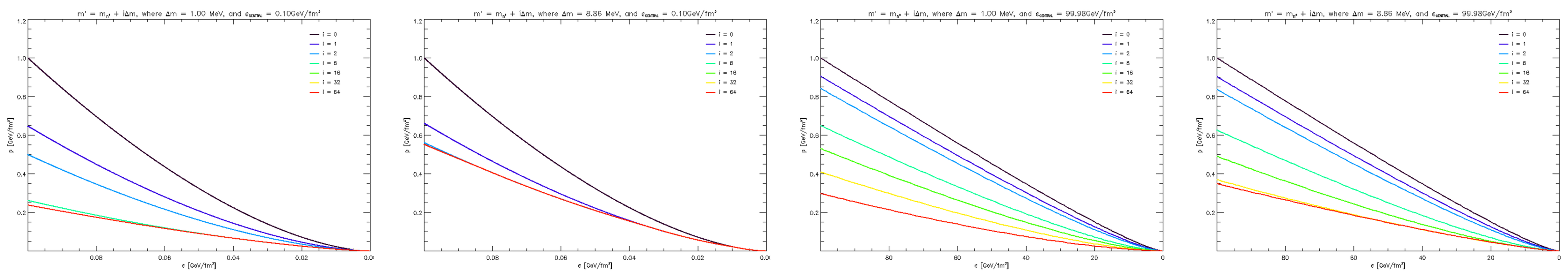
Results

- 1+3D multi component Fermi-gas ⇨ 1+4D one component Fermi-gas with multiple extcitations, *i*
- The simplest equation of state for neutron star models is the non-interacting fermi matter:

$$p_i = -\frac{\partial\Omega|_{T=0}}{\partial V} = \frac{g_i}{24\pi^2} \times \left[\mu_i\sqrt{\mu_i^2 - m_i^2}\left(\mu_i^2 - \frac{5}{2}m_i^2\right) - \frac{3}{2}m_i^4 \ln\frac{m_i}{\mu_i + \sqrt{\mu_i^2 - m_i^2}}\right]$$

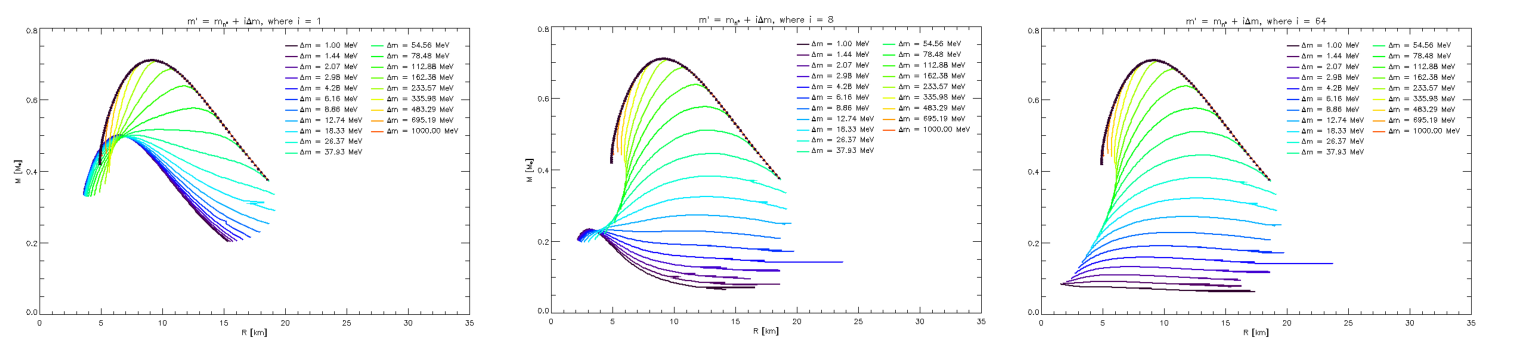
$$\epsilon_i = \frac{g_i}{(2\pi)^3}\int_0^{k_F} \varepsilon_i \mathrm{d}^3\mathbf{k}|_{T=0} = \frac{g_i}{8\pi^2} \times \left[\mu_i\sqrt{\mu_i^2 - m_i^2}\left(\mu_i^2 - \frac{1}{2}m_i^2\right) + \frac{m_i^4}{2}\ln\frac{m_i}{\mu_i + \sqrt{\mu_i^2 - m_i^2}}\right]$$

- The statistics of onehalf-spin particles follow the Fermi–Dirac statistics and without interaction only the number of different degrees of freedoms (flavors) and/or the excited mass gap can tune the equation of state. By introducing many-flavor we can soften the EoS, however the excited-mass gap able to shift the opening of an excited state to higher chemical potentials.



The equation of state, p(ε). The ratio of the excited EoS to the pure ground state EoS. Left panels for ε = 0.1 GeV/fm³, i ∈ [1:64] with 1 MeV and 8.86 MeV mass gap, right panels for ε ~ 100 GeV/fm³, i ∈ [1:64] with 1 MeV and 8.86 MeV mass gap, respectively.

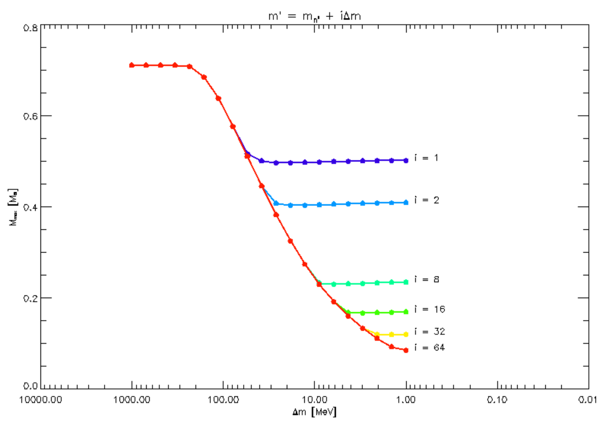
- Including the above EoS to the TOV equation, we can calculate the mass-radius relation, M(R)



The mass-radius relation of the many-flavour compact stars in non-interacting fermion matter, calculated with various mass gap, Δm ∈ [1.0 MeV:1 GeV] at excitations i= 1, 8, and 64 from left to right, respecively.

Summary and Conclusion

- Connecting the Kaluza–Klein compact star with a many-flavor fermion star, we obtained:
  - Many flavors can be interpreted in this framework as one extra compactified spatial dimension with various, more-and-more massive hadron state excitations.
  - The effect of increasing the degrees of freedom was investigated on the equation of state and in connection with the mass-radius relation, M(R).
  - The structure of the excited states (Kaluza-Klein ledder) determines the EoS structure and effect weakly on the M(R) relation.
- Observation of compact stars by multi-wavelength measurements had opened new possibilities for the measurement of macroscopical observables of neutron stars. Especially, gravitational wave interferometry led us to test the law of gravity itself. Alternative gravitational theories can be tested by gravitational wave detectors (Raffai et al. 2017) and predictions can be made for the parameter of the non Newtonian gravitational filed.



The maximal mass of relation of compact stars. The M_{max} (Δm) diagram clearly presents that increasing the Δm , the maximum mass of the star is getting larger, since the EoS becomes stiffer. As getting Δm closer to 200MeV the M_{max} increases and saturates to a maximum 0.7$M_\odot$, independently of the excitation number, i. As decreasing the Δm (softens the EoS), M_{max} also saturates, but results smaller-and-smaller values depending on the possible degrees of freedom.

Acknowledgements

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Related publications

- *Int.J.Mod.Phys. A31 (2016) no.28&29, 1645031*
- *Acta Phys.Polon.Supp. 10 (2017) 827*